Theory of Computation

Spring 2024, Homework #4

Due: June 4, 2024

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1. (20 pts) Recall that $HALT_{TM} = \{\langle M, w\rangle \mid M$ is a TM and M halts on input $w\}$. Prove that $HALT_{TM}$ is NP-hard. (Hint: you may use the fact that if $L \in NP$, there exists a polynomial time TM M and a polynomial $p : N \to N$ such that for every $x \in \{0,1\}^*, x \in L \Leftrightarrow \exists u \in \{0,1\}^{p(|x|)}$ s.t. $M(x,u) = 1$ (i.e., accepts).)

2. Prove formally that NP is closed under reversal by

   (a) (10 pts) construct a nondeterministic TM;

   (b) (10 pts) construct a polynomial time verifier.

3. (20 pts) Let $L_1 = \{\langle M\rangle \mid M$ is a TM and $\langle M\rangle \notin L(M)\}$. Prove that A_{TM} cannot be many-one reducible to L_1 (i.e., $A_{TM} \nleq_m L_1$).

4. (20 pts) Prove that $L_2 = \{\langle M\rangle \mid M$ is a TM and M halts on all palindromes. $\}$ is not Turing-recognizable.

5. (20 pts) Let P be a property over TM descriptions. Assume that P satisfies the following properties:

   - For any TMs M_1 and M_2, where $L(M_1) = L(M_2)$, $\langle M_1\rangle \in P$ if and only if $\langle M_2\rangle \in P$.
   - There are TMs M_{in} and M_{out}, where $\langle M_{in}\rangle \in P$ and $\langle M_{out}\rangle \notin P$, such that $L(M_{in}) \subset L(M_{out})$.

   Prove that L is not Turing-recognizable.