Theory of Computation

Spring 2024, Homework #3

Due: May 7, 2024

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1. (30 pts) Determine whether or not each of the following languages is decidable. If it is decidable, describe a Turing machine that decides it. If it is not decidable, show this using a reduction from a problem shown (in class notes) to be undecidable.

   (a) $L_1 = \{\langle M, w\rangle \mid M \text{ ever moves left while computing } w\}$.

   (b) $L_2 = \{\langle M, w\rangle \mid M \text{ ever moves left three times in a row while computing } w\}$.

2. (10 pts) Let $EQ_{CFG} = \{\langle G_1, G_2\rangle \mid G_1, G_2 \text{ are context-free grammars, and } L(G_1) = L(G_2)\}$. Prove that EQ_{CFG} is co-Turing-recognizable.

3. (30 pts) Let $\mathit{spaceBound}(M, w, n)$ be true iff Turing maching M accesses at most n tape squares when run with input w. Let

$$B_1 = \{\langle M\#w\rangle \mid \mathit{spaceBound}\,(M, w, 2^{|w|})\}$$
$$B_2 = \{\langle M\rangle \mid \forall w\ \mathit{spaceBound}\,(M, w, 2^{|w|})\}$$

   One of these languages is decidable and one is not. Determine which is which and justify your answers. Hint: You might consider using computation history in your solution.

4. (30 pts) For each of the following variants of the Post Correspondence Problem (PCP), say if it is decidable or not. Justify your answer by describing a decider, or by reducing from (standard) PCP.

   (a) $PCP_\star = \{\langle P\rangle \mid P \text{ is in PCP and every domino in } P \text{ is a } \star\text{-domino }\}$. A $\star$-domino is a domino where both the top and bottom strings begin with $\star$. For example, $\left[\frac{\star ab \star c}{\star}\right]$ is a $\star$-domino, but $\left[\frac{a\star}{\star}\right]$ is not.

   (b) $PCP_1 = \{\langle P\rangle \mid P \text{ is in PCP and every domino in } P \text{ is a one-symbol domino }\}$. A one-symbol domino is a domino where the top and bottom strings have length at most one. For example, $\left[\frac{a}{\epsilon}\right]$, $\left[\frac{\epsilon}{b}\right]$, and $\left[\frac{a}{b}\right]$ are all one-symbol dominos. Hint: If $t_{i_1}\cdots t_{i_k} = b_{i_1}\cdots b_{i_k}$, then $|t_{i_1}\cdots t_{i_k}| = |b_{i_1}\cdots b_{i_k}|$. So a match must have the same number of ϵ's in both the top and the bottom strings.