Theory of Computation

Spring 2024, Homework #2

Due: April 9, 2024

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1. (20 pts) Consider the following language: $L = \{a^n b^m c^k \mid n = m \text{ or } m \neq k\}$.
   (a) Design a context-free grammar for L. Explain (informal way is sufficient) why your grammar works.
   (b) Find a pushdown automaton recognizing L. You can either provide the formal definition $(Q, \Sigma, \Gamma, \delta, q_0, F)$, or draw the PDA (where the transitions must be clearly defined).

2. (20 pts) Use pumping lemma to prove that the language $C = \{w \mid \exists i, j \geq 0, w = a^i b^j c^i d^j\}$ is not context-free.

3. (20 pts) Show that the class of CFLs is not closed under the operation *shuffle*.

4. (20 pts) If A and B are languages, define $A \Diamond B = \{xy \mid x \in A, y \in B, |x| = |y|\}$. Show that if A and B are regular languages, then $A \Diamond B$ is a CFL. To this end, let $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ be DFA accepting A and B, respectively. Construct in a precise manner a PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ to accept $A \Diamond B$.

5. (20 pts) Convert the following CFG (over $\{0\}^*$) into an equivalent CFG in Chomsky normal form. Show the steps you took (as illustrated in lecture notes).
   $S \to BSB \mid B \mid \epsilon$
   $B \to 00 \mid \epsilon$