

Theory of Computation

Spring 2023, Homework # 3

Due: June 6, 2023

- (20 pts) Let $S_{TM} = \{\langle M \rangle \mid \text{Turing machine } M \text{ accepts } w \text{ whenever it accepts } w^R\}$. Show that S_{TM} is undecidable.
- (20 pts) Show that a language A is Turing-recognizable if and only if there exists a Turing-decidable language B such that $A = \{x \mid \text{there exists } y \text{ such that } \langle x, y \rangle \in B\}$
- (20 pts)
 - If $A \leq_m B$ and B is a regular language, does that imply that A is also a regular language? Justify your answer.
 - Give an example of an undecidable language B such that $B \leq_m \overline{B}$.
- (20 pts) Let A and B be two disjoint languages. Say that language C separates A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.
- (20 pts) Prove formally that NP is closed under *union*, *intersection*, *concatenation*, and *Kleene star* (i.e., $L \in NP \Rightarrow L^* \in NP$).