Theory of Computation

Spring 2023, Homework # 2

Due: May 9, 2023

1. (20 pts) Prove that $\{a^n b^j c^k \mid k = jn\}$ is not context-free. Show your work in detail. Hint: Consider $a^m b^m c^{m^2}$, where m is the pumping constant.

2. (20 pts) Let $\Sigma = \{0, 1, +, =\}$ and $ADD = \{x = y + z \mid x, y, z$ are binary integers, and x is the sum of y and $z\}$. Show that ADD is not regular.

3. (20 pts) Let A be the language $\{a^n b^n \mid n \geq 0\}$ and let $B = \overline{A}$, i.e., B=$\{a, b\}^* \setminus A$. Use closure of the context-free languages under union to show B to be context-free. You can express B as the union of three context-free languages, one of which is $\overline{a^* b^*}$. In your answer, you must show the three languages to be context-free.

4. (30 pts) A context-free grammar is *linear* if every rule is of the form $A \to u$ or $A \to uBv$ with $A, B \in V$ (the set of nonterminals) and $u, v \in \Sigma^*$. A language is linear if it is generated by a linear grammar.

   (a) (15 pts) Prove the following pumping lemma for linear languages: Let L be a linear language. Then there is a constant p such that for any $w \in L$ with $|w| \geq p$, we can write w as $uvxyz$ such that $|uvyz| \leq p$, $|vy| \geq 1$, and $uv^i xy^i z \in L$ for all $i \geq 0$. Note that the difference between this and the one for CFLs is that $|uvyz| \leq p$ here, instead of $|vxy| \leq p$.

   (b) (15 pts) Use the pumping lemma to prove that $L = \{a^n b^{2n} a^n \mid n \geq 0\}$ is not linear.

5. (10 pts) Let B and D be two languages. We write $B \propto D$ if $B \subseteq D$ and D contains infinitely many strings that are not in B. Show that, if B and D are two regular languages where $B \propto D$, then we can find a regular language C such where $B \propto C \propto D$. Hint: Consider $D \setminus B$, which is also regular.