Theory of Computation

Spring 2023, Homework # 1

Due: April 6, 2023

1. (30 pts) Decide whether each of the following languages is regular or not. Give a convincing proof. Let $\#_0(x)$ and $\#_1(x)$ be the numbers of 0s and 1s, respectively, in string $x \in \{0,1\}^*$.

   (a) $\{x \in \{0,1\}^* \mid \#_0(x) > 0, \#_1(x) > 0,\ GCD(\#_0(x), \#_1(x)) = 1\}$, where GCD denotes the greatest common divisor.

   (b) $\{w \in \{0,1\}^* \mid (\#_0(w)\ mod\ 3) = (\#_1(w)\ mod\ 3)\}$.

2. (10 pts) Express the set $\{0^n 1 0^{n-1} 1 0^{n-2} 1 \cdots 1000100101 \mid n \geq 1\}$ in terms of "$\cap$", "$\cup$", "$*$", "$\cdot$" and the set $\{0^{i+1}10^i1 \mid i \geq 1\}$. In this problem, you may use regular expressions such as $\epsilon, 01, 0^*1, ...$ in your expression.
   (**Hint:** Consider $(0^*1 \cdot \{0^{i+1}10^i1 \mid i \geq 1\}) \cap (\{0^{i+1}10^i1 \mid i \geq 1\} \cdot 0^*1)$. What does the language look like?)

3. (15 pts) Prove that the language $\{0^n 1 0^m 1 0^{max(m,n)} \mid n, m \geq 1\}$ is not regular.

4. (15 pts)Let $L_1 = \{ww \mid w \in \{0,1\}^*\}$ and $L_2 = \{0^n 1^n \mid n \geq 1\}$. Express L_2 in terms of L_1 using morphism, inverse morphism, and intersection with regular sets.
   (**Hint:** Consider a morphism $h_1(0) = 0, h_1(1) = 1, h_1(\tilde{0}) = 0$, where $\tilde{0}$ is a new symbol. First figure out what $h_1^{-1}(00010001) \cap 0^*1\tilde{0}^*1$ is. Then you should be able to complete the proof. You may need another morphism to complete the work.)

5. (30 pts) Let $\Sigma = \{0,1\}$, and let $L = \{x0a \mid x \in \Sigma^*, a \in \Sigma\}$, i.e., L is the set of all bit strings whose second-to-last character is a 0.

   (a) What are the equivalence classes of $\equiv_L$? Enumerate these classes, and use it to construct a minimum-sized DFA accepting L. Recall that $x \equiv_L y$ iff $\forall z \in \Sigma^*, xz \in L \Leftrightarrow yz \in L$.

   (b) What is the smallest (in terms of number of states) NFA that accepts L? Show that there are at least three nonisomorphic NFAs of this minimum size that all accept L.