Theory of Computation

Final Exam. 2007

NAME: ....

Student ID.: ....

1 (72 pts)) True or false? (Score = Right - $\frac{1}{2}$Wrong.) Mark 'O' for true; '$\times$' for false.

1. .....*O*..... **$\{a^i | i$ is prime$\}$ is not context free.**
2. .....*X*..... **$\{(a^n b)^n | n \geq 1\}$ is context free.**
3. .....*X*..... **$\{(a^n b)^m | m, n \geq 1\}$ is context free.**
4. .....*O*..... **If L_1 is context free and L_2 is regular, then L_1/L_2 is context free. (Note that $L_1/L_2 = \{x \mid \exists y \in L_2, xy \in L_1\}$)**
5. .....*X*..... **If L_1/L_2 and L_1 are context free, then L_2 must be recursive.**
6. .....*X*..... **If L_1 and $L_1 \cup L_2$ are context free, then L_2 must be context free.**
7. .....*O*..... **If L_1 is context free and L_2 is regular, then $L_1 - L_2$ is context free.**
8. .....*X*..... **If L_1 is regular and L_2 is context free, then $L_1 - L_2$ is context free.**
9. .....*O*..... **If L_1 is regular and L_2 is context-free, then $L_1 \cap L_2$ must be a CFL.**
10. .....*O*..... **If L_1 and L_2 are CFLs, then $L_1 \cup L_2$ must be a CFL.**
11. .....*O*..... **If L is context free, then L^R (=$\{x^R | x \in L\}$) is also context free.**
12. .....*X*..... **Nondeterministic and deterministic versions of PDAs are equivalent.**
13. .....*O*..... **If a language L does not satisfy the conditions stated in the pumping lemma for CFLs, then L is not context-free.**
14. .....*X*..... **Every infinite set of strings over a single letter alphabet Σ (=$\{a\}$) contains an infinite context free subset.**
15. .....*X*..... **Every infinite context-free set contains an infinite regular subset.**
16. .....*O*..... **A language can be accepted by a nondeterministic pushdown automaton iff it can be generated by a context-free grammar.**
17. .....*O*..... **Right-linear grammars are special cases of context-free grammars.**
18. .....*X*..... **If both L and $\bar{L}$ are context-free, then L must be regular.**
19. .....*O*..... **There is a language L which is context-free but not regular such that $\bar{L}$ is also context-free.**
20. ....*O*...... **$\{xxxx | x \in \{0,1\}^*\}$ can be accepted by a deterministic 2-counter machine.**
21. .....*O*..... **Given a TM M whose tape head can move left, right, or stay stationary, the problem of determining whether M ever executes a stationary move is undecidable. (A stationary move is a transition without moving the tape head.)**
22. .....*O*..... **Given a TM M, the problem of determining '$L(M) = \emptyset$?' is undecidable.**

23. .....O..... Given two languages L_1 and L_2, if $L_1 \leq_m \bar{L_2}$, then $\bar{L_1} \leq_m L_2$. ($\leq_m$ denotes many-one reduction.)

24. .....O..... If L_1 and L_2 are r.e., so is $L_1 L_2$.

25. .....O..... If L_1 and L_2 are recursive, so is $L_1 - L_2$.

26. .....X..... The language $\{< M, x >|$ TM M does not accept input $x\}$ is r.e. ($< M, x >$ denotes the encoding of the pair M, x.)

27. .....X..... There exists a language L such that L is context free but $\bar{L}$ is not recursive.

28. .....O..... Given a PDA M, the problem of determining whether M accepts an infinite language is decidable.

29. .....X..... Given two PDA M_1 and M_2, the problem of determining whether $L(M_1) \cap L(M_2) = \emptyset$ is decidable.

30. .....X..... Given two regular languages L_1 and L_2, the problem 'Is $L_2 - L_1 = \emptyset$?' is undecidable.

31. .....X..... Let L_1 be regular and L_2 recursively enumerable. Then $L_1 \cap L_2$ is always recursive.

32. .....X..... The union of infinitely many recursive languages is an r.e. language.

33. .....X..... Given an input x and a multi-tape DTM M, the problem of determining whether M ever reads x's right-most symbol is decidable.

34. .....X..... The family of languages accepted by deterministic TMs is closed under complement.

35. .....O..... Given a TM M and an input w, the problem of determining whether M (on input w) enters some state more than 100 times is decidable.

36. .....X..... Every infinite subset of an infinite non-regular language is non-regular.

37. .....O..... If L_1 and L_2 are context-free languages, then $L_1 \cap L_2$ must be a recursive language.

38. .....X..... If L_1 and L_3 are r.e. languages and $L_1 - L_2 = L_3$, then L_2 must be an r.e. language.

39. .....X..... Let L_1 and L_2 be two languages over Σ. If $L_1 \leq_p^m L_2$ and $L_2 \leq_p^m L_1$, then $L_1 = L_2$. ($\leq_p^m$ denotes polynomial-time many-one reduction.)

40. .....X..... Given a recursive language L, the problem of determining whether $L = \emptyset$ is decidable.

41. .....X..... $\{< M > | L(M)$ is regular, M is a TM$\}$ is recursive. ($< M >$ denotes the encoding of TM M.)

42. .....X..... $\{L \mid L = \Sigma^*$ or $L = \emptyset\}$ is a trivial property of r.e. sets.

43. .....O/X..... Given a TM M and an input x, it is decidable whether M ever makes two consecutive right-moves (i.e., a right-move followed by a right-move immediately) during the course of its computation on input x.

44. .....X..... Given a TM M and a symbol $x \in \Sigma$, it is decidable whether M (starting on a blank tape) ever writes x on its tape.

45. .....O..... Every primitive recursive function is a total function.

46. .....O..... Ackermann's function is a total recursive function.

47. .....O..... Nondeterministic 1-counter machines are less powerful than deterministic 2-counter machines.

48. .....X..... **Given a context-free language L_1 and a recursive language L_2, it is undecidable whether $L_1 \subseteq L_2$.**

49. .....X..... **Rice's theorem is a useful tool for showing a language to be recursive.**

50. .....X..... **If L and L^R (the reversal of L) are both in r.e., then L must be recursive.**

51. .....X..... **Given a recursive set L and a regular set R, it is decidable whether $L \subseteq R$.**

52. .....X..... **Given a recursive set L and a regular set R, it is decidable whether $R \subseteq L$.**

53. .....O..... **Given a nondeterministic finite automaton M it is decidable whether the language accepted by M is finite or not.**

54. .....O..... **The L_u language (i.e., the universal language)is many-one reducible to the PCP language (the language associated with the Post correspondence problem).**

55. .....O..... **Given a left-linear grammar G, it is decidable whether $L(G) = \Sigma^*$.**

56. .....O..... **Recursive languages are closed under Kleene star (i.e., if L is recursive, so is L^*).**

57. .....O..... **Recursively enumerable languages are closed under Kleene star.**

58. .....O..... **The function $f(n) = 2^{f(n-1)}, n \geq 1; f(0) = 1$ is primitive recursive.**

59. .....X..... **For every language $L \subseteq 0^*$, L is always r.e.**

60. .....X..... **Every total function $f : N \rightarrow N$ is a recursive function. (f is total if $f(x)$ is defined for every $x \in N$.)**

61. .....X..... **With respect to a given input, checking whether a C program terminates or not is decidable.**

62. .....O..... **The language $\{a^n b^m c^n d^m \mid m, n \geq 1\}$ can be accepted by a deterministic TM in polynomial time (i.e., in P).**

63. .....O..... **$\{(a^i b^i)^j \mid i, j \in N\}$ is in P.**

64. .....O..... **The class of NP languages is closed under intersection.**

65. .....O..... **The class of NP languages is closed under concatenation.**

66. .....O..... **For every language $L \in P$, $L \leq_p^m 3SAT$.**

67. .....X..... **If $L = \bigcup_{i=1}^{\infty} L_i$, and each $L_i \in NP$, then $L \in NP$.**

68. .....X..... **Given a context-free grammar G and a word x, the problem 'Is $x \in L(G)$?' is NP-complete.**

69. .....X..... **If $\{ww^R \mid w \in \Sigma^*\}$ is solvable in polynomial time, then $P = NP$. (w^R denotes the reversal of word w.)**

70. .....X..... **If $L_2 \subseteq L_1$, and L_2 is NP-hard, then L_1 must be NP-hard as well.**

71. .....X..... **The minimum spanning tree problem is NP-complete.**

72. .....X..... **If some NP-complete language is solvable in polynomial time, then the PCP problem becomes solvable (i.e., recursive).**

2. (20 pts) Let L_1 and L_2 be languages in the respective language class, and let R be a regular language, and x be a given word over alphabet Σ. Choosing from among (D) decidable, (U) undecidable, (?) open problem, categorize each of the following decision problems. No proofs are required. No penalty for wrong answer.

Language class / Problem	regular	context-free	recursive	r.e.
$L_1 \cup L_2 = \Sigma^*$?	**D**	**U**	**U**	**U**
$x \in L_1$?	**D**	**D**	**D**	**U**
$R \subseteq L_1$?	**D**	**U**	**U**	**U**
$L_1 - R = \emptyset$?	**D**	**D**	**U**	**U**
$\exists y \in L_1, \lvert y\rvert \leq 5$?	**D**	**D**	**D**	**U**

($\lvert y\rvert$ denotes the length of y.)

3. (8 pts) For each $n \in N$ (the set of natural numbers), let C_n be a subset of NP which is closed under polynomial-time many-one reduction (i.e., if $L_1 \leq_p^m L_2$ and $L_2 \in C_n$, then $L_1 \in C_n$). Assume also that each $C_n \overset{\subset}{\neq} C_{n+1}$, and let $C = \bigcup_{n=0}^{\infty} C_n$. Show that $C \neq NP$. Give a brief but convincing argument.

(Proof sketch)

It is known that NP contains some complete languages such as 3SAT. If $C = NP$, then 3SAT $\in C_j$, for some j. Due to the definition of "completeness", $\forall L \in C(= NP)$, $L \leq_p^m 3SAT$. This implies that $\forall L' \in C_{j+1}(\subseteq C)$, $L' \leq_p^m 3SAT$; hence, $L' \in C_j$. We have that $C_{j+1} = C_j$ – a contradiction.