Theory of Computation

Final Exam.

Spring 2001 (YEN)

1. (15 pts) Let $DOUBLE-SAT = \{\langle\phi\rangle \mid \phi \text{ has at least two satisfying assignments}\}$. Prove in detail that $DOUBLE-SAT$ is NP-complete.

2. (20 pts) **True or False?** You must provide a short yet convincing explanation for your answer, in order to receive full credit.

   (a) If PH (the *polynomila time hierarchy*) has a complete problem, then $P = NP$.

   (b) The language $ALL_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ is decidable. (Σ is the alphabet and $\langle A\rangle$ denotes the encoding of A.)

   (c) If $A \leq_m^P B$ (A is polynomial-time many-one reducible to B) then $A \leq_T^P B$ (A is polynomial-time Turing reducible to B).

   (d) If $A \leq_T^P B$, then $A \leq_T^P \bar{B}$. ($\bar{B}$ is the complement of B.)

   (e) If $L_2 \subseteq L_1$ and L_2 is NP-hard, then L_1 must also be NP-hard.

3. (15 pts) For any class of langauges $\mathcal{C}$, $co-\mathcal{C} = \{\bar{L} \mid L \in \mathcal{C}\}$. Prove that $NP \cup co-NP \subseteq P^{SAT}$ (P^{SAT} is the class of languages accpeted by polynomial time DTMs using SAT as oracles). Show your proof in sufficient detail. (Hint: Recall that SAT is NP-hard.)

4. (15 pts) Is it true that '$L \in NP$ implies $L^* \in NP$'? Justify your answer in sufficient detail. (Recall that $L^* = \{x_1x_2\cdots x_k \mid k \geq 0, x_1, x_2, ..., x_k \in L\}$.)

5. (15 pts) Draw the containment relationships for the following complexity classes: *DL, NL, P, NP,* $\Sigma_i^P, \Delta_i^P, \Pi_i^P (\forall i, 0 \leq i)$, *PH, PSPACE, BPP, recursive, r.e., EXPTIME, EXPSPACE,* P^{NP}.

6. (15 pts) Prove that RP is closed under intersection (i.e, if $A, B \in RP$, so is $A \cap B$). (Recall that $A \in RP$ if there is a polynomial-time probabilistic TM accepting A with zero error probability for every $x \notin A$.)

7. (15 pts) Define the following two PCPs.

   (a) *Post correspondence problem.* What does this problem mean?

   (b) *Probabilistically checkable proof.* What does the class $PCP(r(n), t(n))$ mean?