Solution to HW3

December 27, 2013

1.(Linear CFG $\rightarrow$ 1-turn PDA) We can assume that every production in $G = (N, A, S, P)$ is of the form $A \rightarrow uBv$ or $A \rightarrow u$, where A, B are non-terminals, and u, v are terminals or empty strings. An 1-turn PDA M that accepts $L(G)$ is described as follows. There is only one state q_1. The bottom-of-stack symbol $\perp = S$ is the start non-terminal symbol of G. Acceptance condition is empty stack. The transitions are: (1) If $A \rightarrow uBv$ is a production, $(q_1, u, A) \rightarrow (q_1, Bv)$, and (2) if $A \rightarrow u$ is a production, $(q_1, u, A) \rightarrow (q_1, \epsilon)$, and (3) $(q_1, u, u) \rightarrow (q_1, \epsilon)$ for each non-terminal u.

1.(1-turn PDA $\rightarrow$ Linear CFG) Please refer to Theorem 2.1 of Ginsburg, S. and Spanier, E. "Finite-Turn Pushdown Automata". SIAM Journal on Control 1966 4:3, 429-453.

2. We follow the hint. L' is clearly not CFL (by an similar argument for proving non-CFL of $\{a^n b^n c^n | n \geq 0\}$ by applying pumping lemma.). To design an NPDA M_3 that accepts L', assuming that L were a DCFL, let M_1 be a DPDA accepting L and M_2 be a copy of M_1. The construction of M_3 is described as follows: (1) Initial state of M_3 is the initial state of M_1. (2) Final states of M_3 is union of the final states in M_1 and M_2. (3) We change all the b in the transitions of M_2 to c. (4) Let S denotes the set of states in M_1 that accepts some $a^i b^i$ for some $i \geq 0$. For every $q \in S$, we add an ϵ-transition $(q, \epsilon, X) \rightarrow (q', X)$ for each $X \in \Gamma$, where q' is the corresponding state of q in M_2. The proof of correctness is quite tedious so we simply omit here.

3.(a) We show that the statement of pumping lemma (p.60 of S-3) holds for L. Let k be any constant greater than 0. For any word z in L of length at least k, we use z_i to denote the i^{th} letter of z. Set $u = v = w = \epsilon$, $x = z_1$, $y = z_2 z_3 \ldots z_{|z|}$. Now $uv^i wx^i y = {z_1}^i z_2 z_3 \ldots z_{|z|}$ must be in L since (1) if $z_1 = a$, clearly $uv^i wx^i y \in \{a^i b^j c^k d^l | j = k = l\}$, (2) if $z_1 \neq a$, clearly $uv^i wx^i y \in \{a^i b^j c^k d^l | i = 0\}$.

3.(b) For all I, let $u = a^{I+1}b^{I+1}c^{I+1}d^{I+1} \in L$. We mark all the c in u. Now for any valid $u = vwxyz$, (1) wxy cannot contains both b and d (if so, wxy would contain more than I marked positions), and (2) w or y contains at least one c (since only c is marked). Therefore vw^mxy^mz must not have $\sharp(b) = \sharp(c) = \sharp(d)$, where $\sharp(s)$ denotes the number of occurrence of s. Therefore, for any $m > 0$, $vw^mxy^mz \notin L$. Hence L is not CFL due to Odgen's lemma.

4.(a) We can assume that L and R share the same alphabet. Let $M_1 = (Q_1, \Sigma, q_1, \delta_1, F_1)$ be a DFA that accepts R, and $M_2 = (Q_2, \Sigma, \Gamma, q_2, \delta_2, \perp, F_2)$ be a PDA that accepts L. We construct a PDA that accepts *shuffle*(L, R) as follows: $M_3 = ((Q_1 \times Q_2), \Sigma, \Gamma, (q_1, q_2), \delta_3, \perp, F_3)$, where $F_3 = \{(q, p) | q \in F_1, p \in F_2\}$. The transitions are (1) $((q, p), a, X) \rightarrow ((\delta_1(q, a)p), X)$ and (2) $((q, p), a, X) \rightarrow ((q, p'), Z')$, where $(p, a, X) \rightarrow (p', Z')$ is a transition in M_2, for all $q \in Q_1, p \in Q_2, a \in \Sigma, X \in \Gamma$. Intuitively, type 1 and type 2 transitions represent moves in M_1 and M_2, respectively.

4.(b) Let $L_1 = \{a^nb^n | n \geq 1\}$, $L_2 = \{c^nd^n | n \geq 1\}$, $L_3 =$*shuffle*(L_1, L_2). If L_3 is context-free, than $L_4 = L_3 \cap L(a^+c^+b^+d^+) = \{a^mc^nb^md^n | m, n \geq 1\}$ must be also context-free since intersection of any context-free language and regular language is also context-free. It is easy to show that L_4 is not context-free by applying pumping lemma, so L_3 is not context-free.