Solution to HW2

November 20, 2013

1. Let L be $\{ww|w \in \Sigma^*\}$. Note that L is regular when $|\Sigma| < 2$. Therefore, from now on we assume $|\Sigma| \geq 2$. We show that the R_L defined by Myhill-Nerode theorem has infinite index. Let $0, 1$ be two letters in Σ. We write s_i to denote 0^i1. For all $1 \leq i, j$, we have $s_is_i \in L$ but $s_js_i \notin L$. Therefore, all s_i belongs to different equivalence classes. R_L must have infinite number of equivalence classes. Hence L is non-regular if and only if $|\Sigma| \geq 2$.

2. L_1 is not regular. We show that the R_{L_1} defined by Myhill-Nerode theorem has infinite index. We write s_i to denote $(01)^i$. For all $1 \leq i, j$, we have $s_i(s_i)^R \in L_1$ but $s_j(s_i)^R \notin L_1$. Therefore, all s_i belongs to different equivalence classes of R_{L_1}. R_{L_1} must have infinite number of equivalence classes. L_2 is regular. It is easy to see that $w \in L_2$ if and only if $|w| \geq 2$ and its first letter and last letter are the same. Therefore, L_2 and be expressed by regular expression $0(0 + 1)^*0 + 1(0 + 1)^*1$.

3. It is easy to see that $w \in A$ if and only if the first letter of w is 1 and the occurrence of 1 in w is at least two. Therefore, A can be described by the regular expression $1(0 + 1)^*1(0 + 1)^*$. For non-regularity of B, we show that the R_B defined by Myhill-Nerode theorem has infinite index. We write s_i to denote 1^i0. For all $1 \leq i < j$, we have $s_i1^j \notin B$ but $s_j1^j \in B$. Therefore, all s_i belongs to different equivalence classes of R_B. R_B must have infinite number of equivalence classes.

4. See the following state transition table.

State	0	1
$\rightarrow *C_1$	C_3	C_5
$*C_2$	C_4	C_6
C_3	C_1	C_2
$*C_4$	C_4	C_6
C_5	C_4	C_6
C_6	C_6	C_6