

Theory of Computation

Fall 2013, Homework # 4

Due: December 23, 2013

- (25 pts) Let $L = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs such that for some input } x, \text{ both } M_1 \text{ and } M_2 \text{ halt on } x \}$. Prove that L is r.e. but not recursive.
- (25 pts) Prove that the following language $\{ \langle M_1, M_2 \rangle \mid L(M_1) \leq_m L(M_2) \}$ is not r.e. (Hint: Reduction from \neg HP.)
- (25 pts) Let A and B be two disjoint languages over alphabet Σ . We say that language C separates A and B if $A \subseteq C$ and $B \subseteq \bar{C}$ (i.e., $B \subseteq (\Sigma^* - C)$). Show that any two disjoint *co-r.e.* languages are separable by some recursive language.
- (25 pts) Suppose there are four languages A, B, C , and D . Each of the languages may or may not be recursively enumerable. However, we know the following about them: $A \leq_m B, B \leq_m C$, and $D \leq_m C$. Below are four statements. Indicate whether each one is
 - CERTAIN to be true, regardless of what problems A through D are.
 - MAYBE true, depending on what A through D are.
 - NEVER true, regardless of what A through D are.

Justify your answers.

- A is recursively enumerable but not recursive, and C is recursive.
- A is not recursive, and D is not recursively enumerable.
- If C is recursive, then the complement of D is recursive.
- If C is recursively enumerable, then $B \cap D$ is recursively enumerable.