Theory of Computation

Fall 2013, Homework # 3

Due: December 9th , 2013

1. (25 pts) A CFG is said to be *linear* if and only if any production of G is of one of the following three forms,

$$A \to wB, \quad A \to Bw \quad \text{or} \quad A \to w,$$

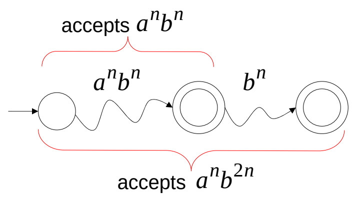
where w is a string of terminals, and A, B are nonterminals. A PDA is single-turn if and only if whenever $(q_0, w_0, Z_0) \overset{*}{\vdash} (q_1, w_1, \gamma_1) \overset{*}{\vdash} (q_2, w_2, \gamma_2) \overset{*}{\vdash} (q_3, w_3, \gamma_3)$ and $|\gamma_2| < |\gamma_1|$, then $|\gamma_3| \leq |\gamma_2|$. That is, once the stack starts to decrease in height, it never increases again. Prove that a language L is generated by a linear grammar iff it is accepted by a single-turn PDA.

2. (25 pts) Prove that $L = \{a^n b^n \mid n \geq 0\} \cup \{a^n b^{2n} \mid n \geq 0\}$ is not a DCFL (deterministic CFL), i.e., it cannot be accepted by any DPDA, where acceptance is defined by final state.
(Hint: by contradiction. Consider the language $L' = L \cup \{a^n b^n c^n \mid n \geq 0\}$. Can you

   (1) show L' to be non-context-free, and

   (2) design an NPDA to accept L', assuming that L were a DCFL?

   Then we have a contradiction. For (2), consider the following figure.)



3. (25 pts) Consider language $L = \{a^i b^j c^k d^l \mid i = 0 \quad or \quad j = k = l\}$.

   (a) Show that the classical pumping lemma fails in showing L to be non-context-free.

   (b) Use Ogden's lemma to show that L is not context-free.

4. (25 pts) Consider the *shuffle* operation discussed in class.

   (a) Show that if L is a CFL and R is a regular language, then *shuffle*(L, R) is a CFL. Hint: start with a PDA for L and a DFA for R.

   (b) Give a counterexample to show that if L_1 and L_2 are both CFL's, then *shuffle*(L_1, L_2) need not be a CFL.