Theory of Computation

Final Exam. 2006

1. (20 pts) Let L_1 and L_2 be languages in the respective language class, and let R be a regular language. Choosing from among **(D) decidable, (U) undecidable, (?) open problem**, categorize each of the following decision problems. No proofs are required. No penalty for wrong answer.

Language class / Problem	regular	context-free	recursive	r.e.
$L_1 = L_2$?	D	U	U	U
$L_1 \cap L_2 = \emptyset$?	D	U	U	U
$L_1 \subseteq L_2$?	D	U	U	U
$L_1 - L_2 = \emptyset$?	D	U	U	U
$L_1 \subseteq R$?	D	D	U	U

2. (28 pts) Answer whether a language class is closed under an operation by filling in the following blanks (28, in all) with one of
Yes (O), No (X), Open problem (?). No penalty for wrong answer.

Language class / Operation	DCFL	CFL	CSL	Recursive	Co-r.e.
Intersection	X	X	O	O	O
Intersection with a regular set	O	O	O	O	O
Complementation	O	X	O	O	X
Concatenation	X	O	O	O	O
Union	X	O	O	O	O
Reversal	X	O	O	O	O

Note: **P** denotes polynomial time. **DCFL** denotes deterministic context-free languages, which are languages that can be accepted by deterministic pushdown automata. **CSL** (context-sensitive languages) are those that can be accepted by LBA (linear-bounded automata). **Co-r.e.** denotes the complement of r.e.

3. (30 pts) True or false (mark **O** for 'true'; **X** for 'false').
(Score=Max{0, Right-$\frac{1}{2}$Wrong}.)

   (a) **X**– Given a TM M, 'M never moves its head left on the blank tape' is a nontrivial property of r.e. sets.

   (b) **O**– Given a TM M and an input x, it is decidable whether M never reads a blank symbol during the course of its computation on input x.

   (c) **X**– Given a TM M and an input x, it is decidable whether M ever visits a given state more than 10 times.

   (d) **O**– Every primitive recursive function is a total function.

   (e) **O**– Ackermann's function is a partial recursive function.

   (f) **O**– Deterministic PDA are less powerful than nondeterministic PDA.

(g) **O**– If L and $\bar{L}$ are both in r.e., then L must be recursive.

(h) **X**–Given an r.e. set L and a regular set R, it is decidable whether $L \subseteq R$.

(i) **X**–Given an r.e. set L and a regular set R, it is decidable whether $R \subseteq L$.

(j) **O**– Given a PDA M it is decidable whether the language accepted by M is finite or not.

(k) **O/X**–The language $\{a^n b^n c^n d^n \mid n \geq 1\}$ can be accepted in polynomial time.

(l) **O**–If some NP-complete language is solvable in polynomial time, then NP=co-NP.

(m) **X**–Every infinite r.e. set contains an infinite context-free subset.

(n) **O**–The halting problem is NP-hard.

(o) **X**–If P=NP, then $DTIME(n^2) = DTIME(2^n)$. (Note: DTIME denotes deterministic time.)

(p) **O**–The PCP language (the language associated with the Post correspondence problem) is in r.e.

(q) **X**–Given a CFG G in Chomsky Normal Form, it is decidable whether $L(G) = \Sigma^*$.

(r) **O**–Every r.e. language can be accepted by a deterministic 2-counter machine.

(s) **O**–There exists a language $L \subseteq 0^*$ which is not in r.e.

(t) **O**–There exists a total function $f : N \rightarrow N$ which cannot be computed by any Turing machine. (f is total if $f(x)$ is defined for every $x \in N$.)

4. (10 pts) Prove that if language A is in r.e. and $A \leq_m \bar{A}$, then A is recursive. ($\bar{A}$ denotes the complement of A.)
Proof sketch Let M be a DTM accepting A. To test whether $x \in A$, accept if M accepts x; reject if M accepts $f(x)$, where f is the many-one mapping associated with the reduction. $\square$

5. (12 pts) Define the following terms precisely:

(a) Church-Turing Thesis

(b) Rice's theorem

(c) Universal Turing machine