Theory of Computation

Final Exam, Spring 2005

1. (15 pts) Prove that the following language is not context-free: $\{a^{(n+1)^2} \mid n \geq 1\}$.

2. (15 pts) Prove that $\{a^i b^j c^k \mid j = max\{i, k\}\}$ is not context-free.

3. (15 pts). Let A and B be two recursively enumerable (r.e.) sets.

   (a) Is $A \cap B$ always recursively enumerable? Why?

   (b) Is $A - B$ always recursively enumerable? Why?

   (c) Is $A \cdot B$ always recursively enumerable? Why? ($\cdot$ denotes concatenation.)

4. (15 pts) Let $A, B, C \subseteq \{0, 1\}^*$.

   (a) Complete the following definition: We say that A is many-one reducible to B, written as $A \leq_m B$, if ...

   (b) Prove that if $A \leq_m B$ and $B \leq_m C$, then $A \leq_m C$

   (c) Is it true that if $A \leq_m B$ and B is regular, then A must be regular as well? Justify your answer.

5. (20 pts) State each of the following terms in a precise manner:

   (a) *Church-Turing Thesis*

   (b) *Rice's Theorem.*

   (c) *Ogden's lemma.*

   (d) *Chomsky normal form for context-free grammars*

   (e) *2-counter machines*

6. (10 pts) Let L_u=$\{< M, w >|$ TM M accepts $w\}$ and L=$\{< M >|$ TM M accepts at least two distinct words$\}$. Show that $L_u \leq_m L$.

7. (10 pts) Show that $\{< M_1, M_2 >| \ L(M_1) \cap L(M_2) = \emptyset\}$ is not recursively enumerable.