Theory of Computation

Final Exam, Spring 2004

1. (15 pts) True or False? (No penalty for wrong answer.)

1) O If A is not recursive and B is a regular language, then $\{x\#y \mid x \in A, y \in B, \# \text{ is a symbol not used in } A \text{ and } B\}$ is not recursive.

2) X {M | M is a TM and L(M) contains *at least* 3800000 elements} is recursive

3) X {M | M is a TM and L(M) contains *at most* 3800000 elements} is recursive.

4) X If A and B are context-free languages, so is A–B (i.e., the difference of A and B).

5) O Every regular language is in P (polynomial time).

6) X A $\le$m B and B is regular, then A is regular. (Here $\le$m denotes the many-one reduction.)

7) O {< M >| M is a DFA and L(M) is finite} is decidable.

8) O If 3CNF-SAT is in P, then P=NP.

9) X Given a PDA M, and a regular expression R,` Is L(R)=L(M)?' is decidable.

10) O $\{w\, w^R \mid w$ is binary string and w^R is the reverse of $w\}$ is in P.

11) O {<M> | M is a Turing machine that accepts the string 1011} is not recursive.

12) X A countably infinite union of recursive sets (i.e., $\cup_{i=1\ldots\infty} L_i$, each L_i is recursive) is always r.e.

13) O The set of all languages over the alphabet {0,1} is uncountable.

14) X There is an NP algorithm to determine whether or not two context-free grammars generate the same language.

15) O $\{a^k b^k c^k d^k e^k f^k \mid k \ge 0\}$ ($\subseteq \{a, b, c, d, e, f\}^*$) can be accepted by a 2-tape DTM in O(n) time.

2. (15 pts) Show the following problem to be NP-complete:

Instance: an undirected graph G = (V,E).

Question: Is there a clique of size $\lceil n/2 \rceil$ (i.e., half of n) in G, where n = |V|, i.e., the number of nodes in V?

(Hint: You may use any of the NP-complete problems discussed in class to do the reduction.)

3. (10 pts) Let G be the following grammar (λ is the empty string)

$$S \rightarrow aB \mid bA$$
$$A \rightarrow a \mid aS \mid bAA$$
$$B \rightarrow \lambda \mid b \mid bS \mid aBB$$

Construct an equivalent CFG in Chomsky Normal Form.

4. (20 pts) Answer whether a language class is closed under an operation by filling in

yes (Y) , no (N), don't know (?)

No penalty for wrong answer. No proofs are required.

Operation/ language class	*Union*	*Intersection*	*complement*	*Concatenation*	*Kleene Star (i.e., *)*
recursive	O	O	O	O	O
r.e.	O	O	X	O	O
Context-free	O	X	X	O	O
NP	O	O	?	O	O

5. (20 pts)

1. State the pumping lemma for context-free languages.
2. Prove that $\{a^p \mid p = n^2 , n \in N\}$ is not context-free.
3. Use the pumping lemma as a tool to design an algorithm to decide whether a given context-free grammar G generates a finite set (i.e., L(G) is finite).

6. (12 pts) Choosing from among **(D) decidable**, **(U) undecidable**, **(?) unknown**, categorize each of the following decision problems. No proofs are required. No penalty for wrong answer.

Problem / Language Class	**Regular**	**Context Free**	**recursive**	**r.e.**
$L = \Sigma^*$?	O	X	X	X
$L = \phi$?	O	O	X	X
$x \in L$, for arbitrary x ?	O	O	O	X

7. (8 pts) Prove formally that if $A \leq_m B$, and $B \leq_m C$, the $A \leq_m C$. That is, the $\leq_m$ relation (i.e., the many-one reduction) is transitive.