Theory of Computation
Fall 2012, Homework # 3

Due: Dec. 22, 2012

1. (15 pts) Let $L = \{< M, w > | M$ ever moves left while computing $w\}$. Is L recursive? Give a convincing argument.

2. (15 pts) Prove that $X = \{< M, w > | M$ is a single-tape TM that never modifies the portion of the tape that contains the input $w\}$ is not recursive. (Note that $< M, w >$ denotes the encoding of M and w together.)

3. (15 pts) If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why? Give a convincing argument.

4. (15 pts) Is the language $L = \{< M > | M$ is a Turing machine with $L(M) \neq \emptyset\}$ recursively enumerabe? Why? Give a convincing argument.

5. (20 pts) Let A and B be two disjoint languages over a common alphabet Σ. Say that language C separates A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. ($\overline{C}$ denotes the complement of C, i.e., $\overline{C} = \Sigma^* - C$.) Show that if A and B are any two disjoint co-r.e. languages, then there exists a recursive language C that separates A and B. (A language L is co-r.e. if its complement $\overline{L}$ is r.e.)

6. (20 pts) Give an example of an undecidable (i.e., non-recursive) subset of 1^*. Prove that your answer is correct.