Theory of Computation

Fall 2012, Homework # 1

Due: Oct. 15, 2012

1. (40 pts) Which of the following statements are correct? If you think a statement is correct, give a proof. If you think it is incorrect, give a counterexample. You may assume the alphabet is $\Sigma = \{a, b\}$.
   (a) If L is regular, then $L' = \{x \mid ax \in L \text{ or } xb \in L\}$ is regular.
   (b) If L is regular, then $L' = \{xx^- \mid x \in L\}$ is regular. Here x^- is x without its last symbol, e.g. $(bab)^- = ba$, $b^- = \epsilon$. (We let $\epsilon^- = \epsilon$.)
   (c) If L is regular, then $L' = \{x \mid xy \in L \text{ for some string } y\}$ is regular.
   (d) If L_1L_2 is regular, then L_2L_1 is regular. (L_1 and L_2 can be any pair of languages, not necessarily regular.)
   (e) If L is regular, then $L' = \{wz \mid zw \in L \text{ for some strings } w, z\}$ is regular.

2. (20 pts) Let $L \subseteq \Sigma^*$ be a language. For two words $x, y \in \Sigma^*$, we write $x \sim_L y$ iff $xv \in L \Leftrightarrow yv \in L$ holds for all $v \in \Sigma^*$.
   (a) (10 pts) Determine the equivalence classes of $\sim_L$ with respect to language $L = L((ab+ba)^*)$. (Hint: consider DFA accepting L.)
   (b) (10 pts) Suppose we define $\equiv_L$ as follows. For two words $x, y \in \Sigma^*$, we write $x \equiv_L y$ iff $uxv \in L \Leftrightarrow uyv \in L$ holds for all $u, v \in \Sigma^*$. How does $\equiv_L$ relate to $\sim_L$? Why?

3. (20 pts) For two languages $L_1, L_2 \subseteq \Sigma^*$, the *left quotient* of L_1 by L_2 is defined by

$$L_2 \setminus L_1 = \{v \in \Sigma^* \mid \exists u \in L_2, uv \in L_1\}.$$

   Given finite automata A_1 and A_2, construct an automaton A such that $L(A) = L(A_2) \setminus L(A_1)$

4. (20 pts) Use pumping lemma ONLY to prove that the following languages are not regular.
   (a) $L_1 = \{0^{n!} \mid n \geq 0\}$, where $n!$ denotes the factorial of n.
   (b) $L_2 = \{0^p \mid p \in N, p \text{ is not a prime }\}$.