Theory of Computation

Homework # 1

Due: Nov. 7, 2011

1. (20 pts) Let L be any subset of 0^*. Prove that L^* is regular.
   (Hint: Consider the greatest common divisor (gcd) of L. Can you first show that the gcd g always exists. Then use g to show the result.)

2. (20 pts) Let L be a language. Define $\frac{1}{2}(L) = \{x \mid \exists y, |x| = |y| \wedge xy \in L\}$. Prove that if L is regular, so is the language $\frac{1}{2}(L)$.

3. (40 pts) For each of the following languages, determine whether the language is regular or not. Why?
   (a) $\{0^m 1^n 0^{m+n} \mid m \geq 1 \wedge n \geq 1\}$
   (b) the set of all strings that do not have three consecutive 0s.
   (c) $\{xwx^R \mid x, w \in (0+1)^+\}$ and x^R is x written backward; for example $(011)^R = 110$.
   (d) $\{xx^R w \mid x, w \in (0+1)^+\}$

4. (10 pts) Use the Myhill-Nerode theorem to show that the set of strings $\{o^i 1^j \mid gcd(i,j) = 1\}$ is not regular.

5. (10 pts) For two words $x, y \in \Sigma^*$, we write $x \sim_L y$ iff $xv \in L \Leftrightarrow yv \in L$ holds for all $v \in \Sigma^*$. Determine the equivalence classes of $\sim_L$ with respect to language $L = \mathcal{L}((aa)^*)$ (i.e., the set of strings containing an even number of a's) over alphabet $\Sigma = \{a, b\}$.