Priority Queues (Heaps)

Priority Queue: Motivating Example

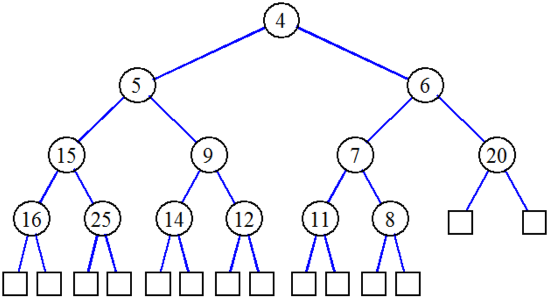
- 3 jobs have been submitted to a printer in the order A, B, C.
  - Job A -100 pages
  - Job B - 10 pages
  - Job C - 1 page

- Average waiting time with FIFO service:
  (100+110+111) / 3 = 107 time units
- Average waiting time for shortest-job-first service:
  (1+11+111) / 3 = 41 time units
- A queue be capable to insert and deletemin?

Priority Queue

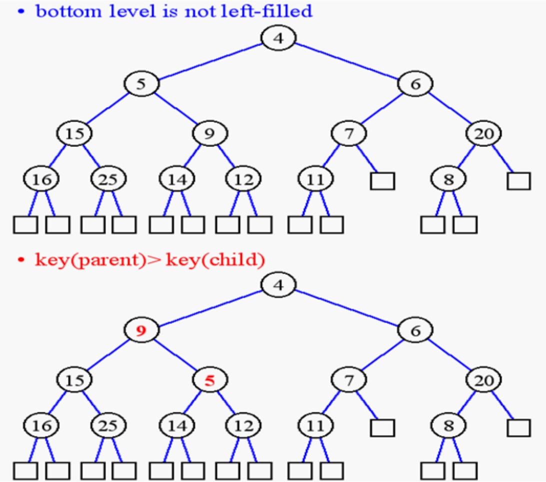
Heaps

- A *heap* is a binary tree T that stores a key-element pairs at its internal nodes
- It satisfies two properties:
  - MinHeap: key(parent) $\leq$ key(child)
    OR MaxHeap: key(parent) $\geq$ key(child)
  - all levels are full, except the last one, which is left-filled, i.e., a complete binary tree.



What are Heaps Useful for?

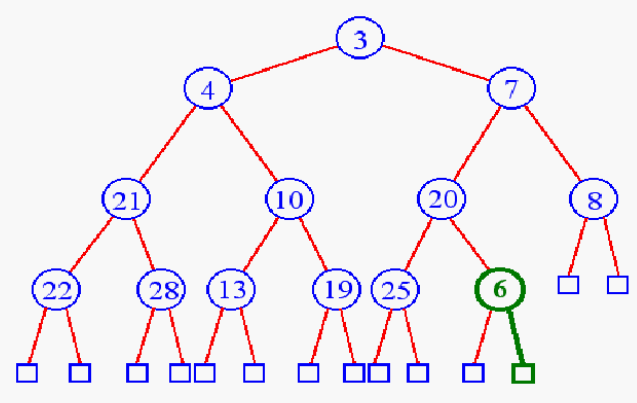
- To implement priority queues
- Priority queue = a queue where all elements have a ″priority″ associated with them
- Remove in a priority queue removes the element with the smallest priority
  - insert
  - removeMin



- A heap T storing n keys has height $h = \lfloor log(n+1) \rfloor$, which is $O(logn)$

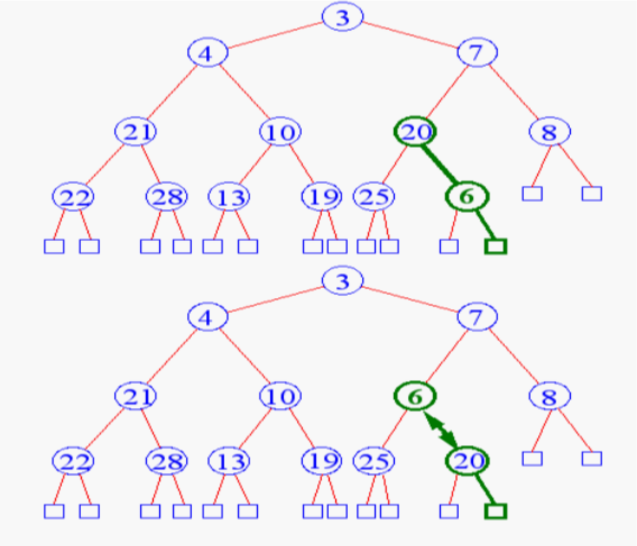
Heap Insertion

- Insert 6 – Add key in next available position

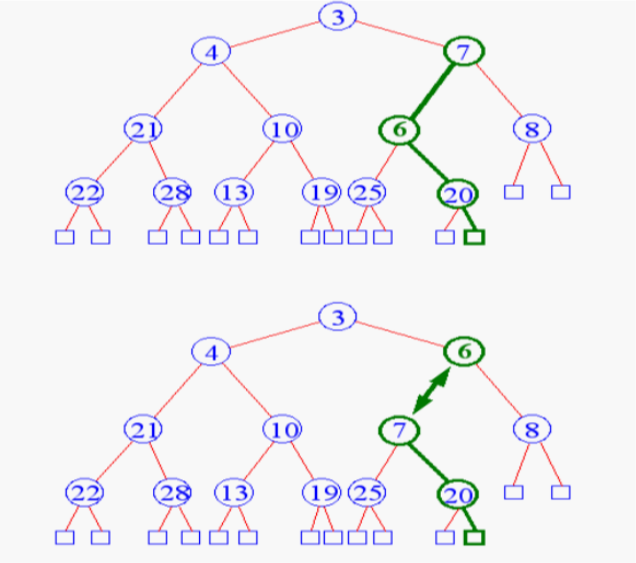


Heap Insertion

- Begin Unheap

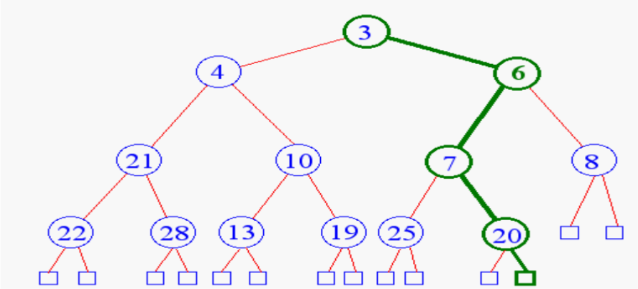


Heap Insertion



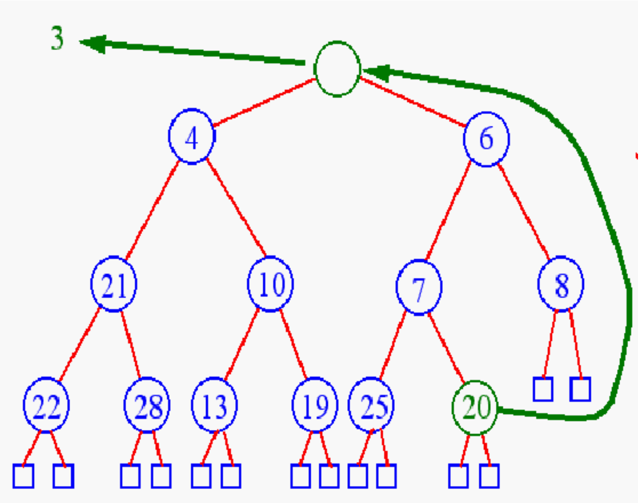
Heap Insertion

- Terminate unheap when
  - reach root
  - key child is greater than key parent



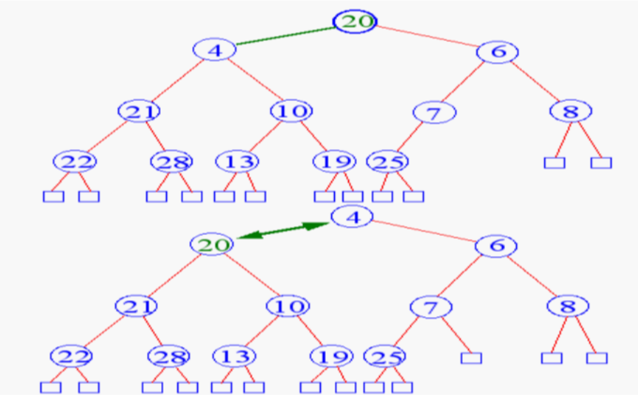
Heap Removal

- Remove element from priority queues? removeMin( )

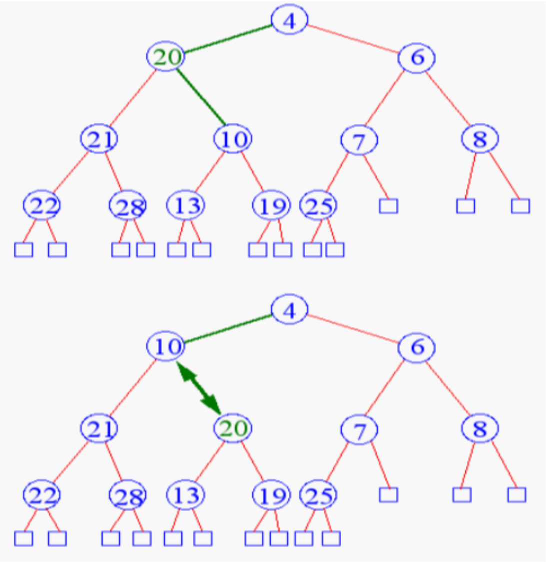


Heap Removal

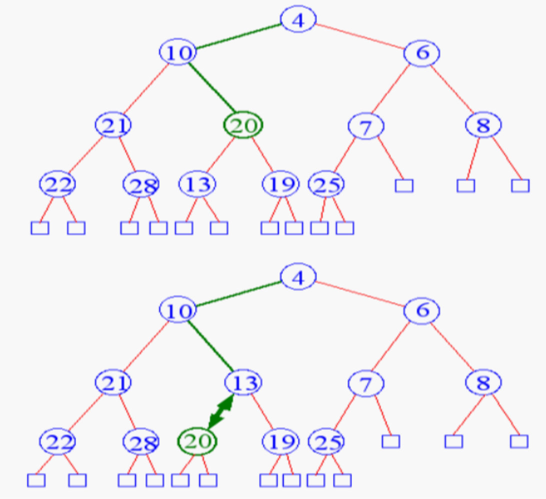
- Begin downheap



Heap Removal

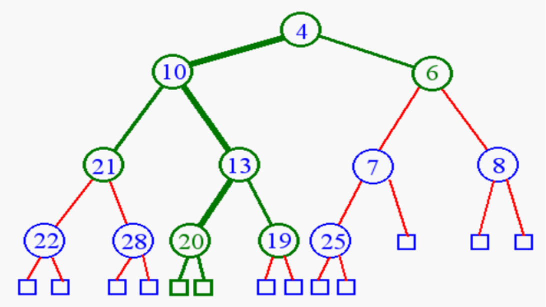


Heap Removal



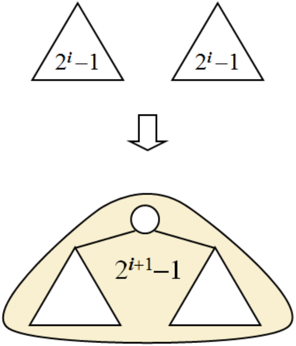
Heap Removal

- Terminate downheap when
  - reach leaf level
  - key child is greater than key parent



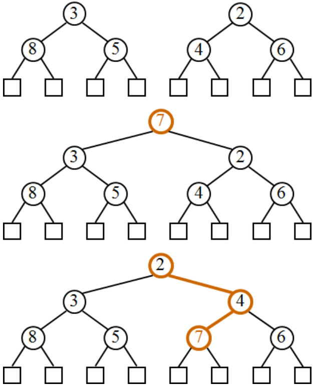
Bottom-up Heap Construction

- We can construct a heap storing n given keys using a bottom-up construction with $log n$ phases
- In phase i, pairs of heaps with $2^i - 1$ keys are merged into heaps with $2^{i+1} - 1$ keys



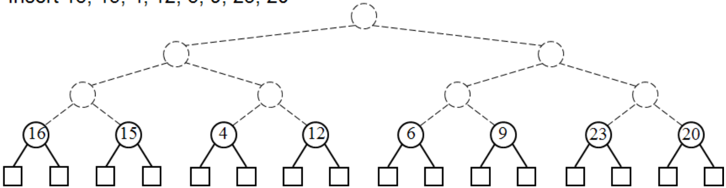
Merging Two Heaps

- We are given two two heaps and a key k
- We create a new heap with the root node storing k and with the two heaps as subtrees
- We perform heapDown to restore the heap-order property

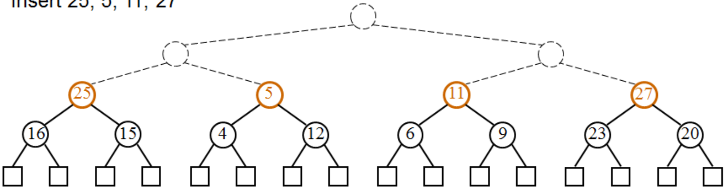


Merging Example

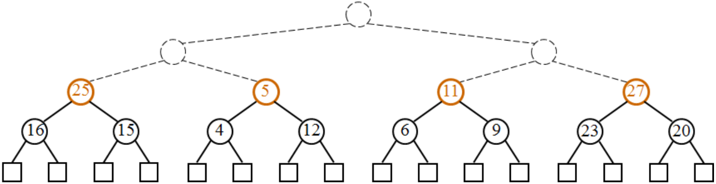
Insert 16, 15, 4, 12, 6, 9, 23, 20



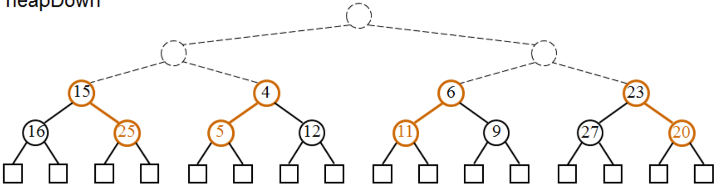
Insert 25, 5, 11, 27



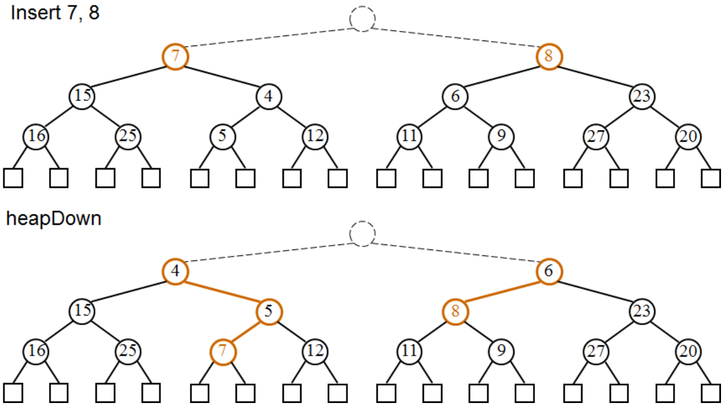
Example (contd.)



heapDown

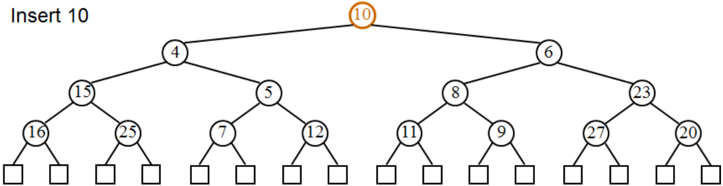


Example (contd.)

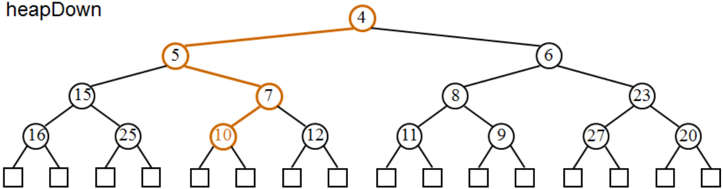


Example (contd.)

Insert 10

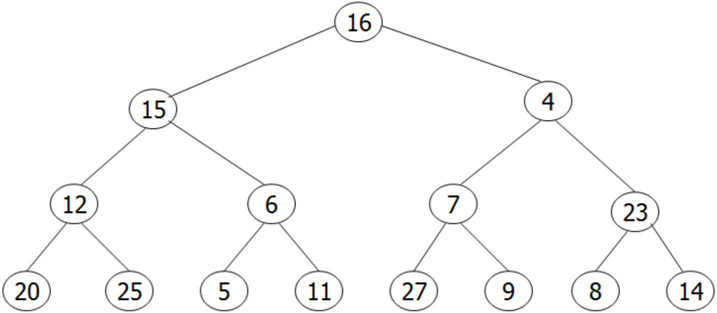


heapDown

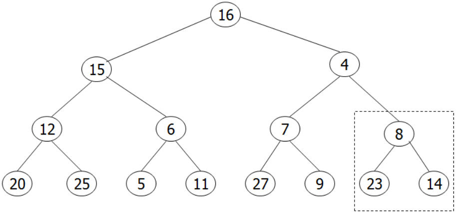


Bottom up Heap Construction

16	15	4	12	6	7	23	20	25	5	11	27	9	8	14



Bottom up Heap Construction



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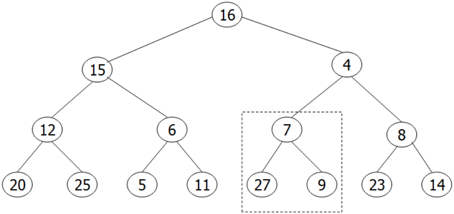

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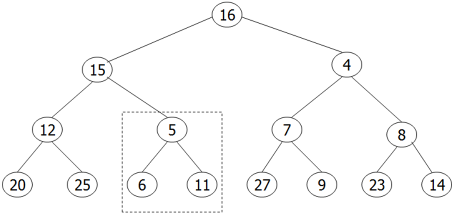
Bottom up Heap Construction



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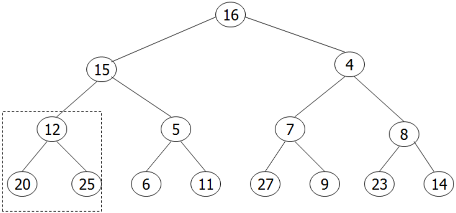
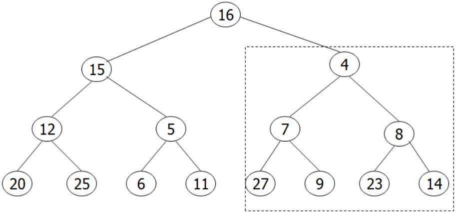


16	15	4	12	5	7	8	20	25	6	11	27	9	23	14

Bottom up Heap Construction



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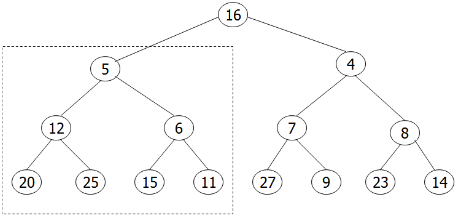
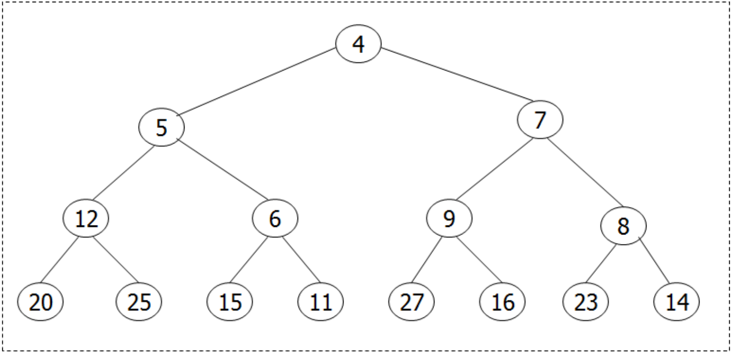


16	5	4	12	6	7	8	20	25	15	11	27	9	23	14

Bottom up Heap Construction



16	5	4	12	6	7	8	20	25	15	11	27	9	23	14

Heap Sorting

- Step 1: Build a heap
- Step 2: removeMin( )
- Running time?