The Solutions of Homework # 1

Problem 1.

Solution:

(a) Ω. (b) O. (c) O. (d) Ω. (e) Ω. (f) O, Ω, Θ. (g) O, Ω, Θ.

$\square$

Problem 2.

Solution:

(A) $T(n) = O(n \log n)$.

$\sum_{i=1}^{\log_2 n} (1 + \sum_{j=1}^{n} 4) = \sum_{i=1}^{\log_2 n} (1 + 4n) = (1 + 4n) \log_2 n = O(n \log n)$.

Note:

(1) The operations we should count within the for-loop are j++ (once), result += 1 (once), and result += 3 * arr[j] (twice) and within the while-loop but the for-loop is i = i/2 (once).

(2) $\sum_{i=1}^{n} c = \sum_{i=0}^{n-1} c$ for all constant c.

(3) If f is a constant function and $n = 2^k$ for some $k \in \mathbb{N}$,

$$f(n) + \cdots + f(\frac{n}{2^i}) + \cdots + f(1) = \sum_{i=1}^{k} f(1) = \sum_{i=1}^{\log_2 n} f(1).$$

$\square$

(B) $T(n) = O(n)$.

$\sum_{i=1}^{\frac{n}{2}} 3 + \sum_{i=\frac{n}{2}+1}^{n} 3 + \sum_{i=1}^{n} 5 = \frac{3n}{2} + \frac{3n}{2} + 5n = O(n)$.

$\square$

(C) $T(n) = O(n^2)$.

$$\sum_{i=1}^{n}(1+\sum_{j=1}^{5}(1+\sum_{k=1}^{n}4)) = \sum_{i=1}^{n}(1+\sum_{j=1}^{5}(1+4n)) = \sum_{i=1}^{n}(1+5+20n) = 6n+20n^2.$$

□

Problem 3.

Solution:

$f_4, f_8, f_5, (f_3, f_6), f_9, f_1, f_{10}, f_2, f_7$ is the list of functions $f_1, \ldots, f_{10}$ in order of their asymptotic growth. The reasons are as follows.

- Since $m^{\log_\alpha n} = n^{\log_\alpha m}$, $f_5(n) = n$, $f_8(n) = n^{\log_{50} 2} = O(n)$, and $f_9(n) = n^2$.
- For every constant $c > 0$, there is no $n_0 \in \mathbb{N}$ such that $(n+1)! \leq c \cdot n!$ for all $n > n_0$. Therefore $f_7 \neq O(f_2)$.

□

Problem 4.

Solution:

The statement is not true. The idea is simple. Let a, b, c, d be four positive numbers. $a \leq b$ and $c \leq d$ does not imply $\frac{a}{c} \leq \frac{b}{d}$.

So, let $f(n), g(n), h_1(n), h_2(n)$ be n^2, n, n^3 and n^3, respectively. Then $f = O(h_1)$ and $g = O(h_2)$. But, however, $\frac{f(n)}{g(n)} = n \neq O(\frac{h_1(n)}{h_2(n)}(=1))$.

□