Data Structures and Programming

Spring 2015, Midterm Exam.

Date: April 28, 2015

⟨ 1 ⟩ (15 pts) Use < and = to order the following functions by asymptotic growth rate:

$(a)4n\log n+\sqrt{n}$ $(b)2^{\log(n^2)}$ $(c)3n+20\log^2 n$ $(d)\log(n^5)$ $(e)n\log n$ $(f)2^{200}$ $(g)\frac{1}{n}$ $(h)3^{\log n}$ $(i)n!$ $(j)\frac{n}{2^n}$

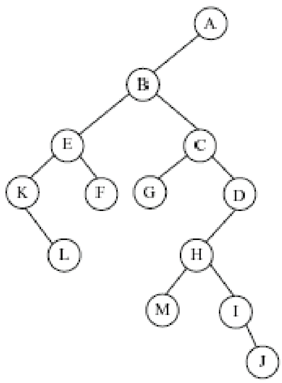
Solution: $(j)\frac{n}{2^n} < (g)\frac{1}{n} < (f)2^{200} < (d)\log(n^5) < (c)3n+20\log^2 n < (a)4n\log n+\sqrt{n} = (e)n\log n < (h)3^{\log n} < (b)2^{\log(n^2)} < (i)n!$

⟨ 2 ⟩ (20 pts) With respect to the tree below, answer the following questions:

(a) (8 pts) Suppose the tree is regarded as a binary tree, give the *preorder, inorder, postorder* and *level-order* traversal sequences.

(b) (7 pts) Suppose the tree is regarded as a binary tree, add threads to make it a threaded binary tree.

(c) (5 pts) Suppose the tree is regarded as an ordered tree, draw its binary tree representation (i.e., Left Child-Right Sibling Representation).

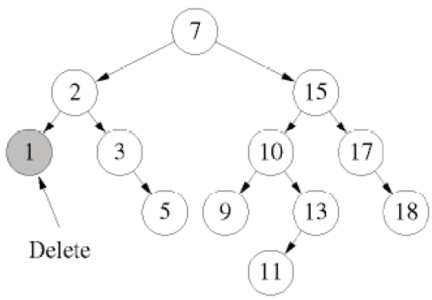


Solution:

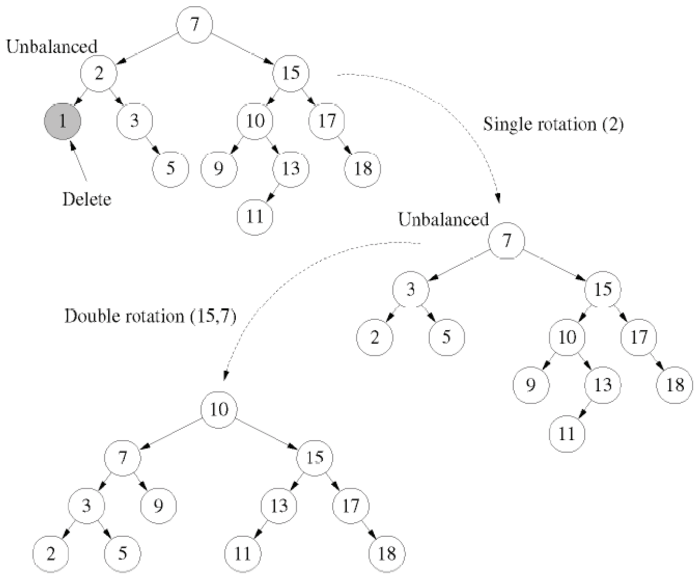
(a) Preorder: A B E K L F C G D H M I J
Inorder: K L E F B G C M H I J D A
Postorder: L K F E G M J I H D C B A
Level-order: A B E C K F G D L H M I J
r

(c)

⟨ 3 ⟩ (15 pts) Given the AVL tree below, show the AVL tree that would result after deleting the key of value 1. Show your derivation in sufficient detail.



Solution:

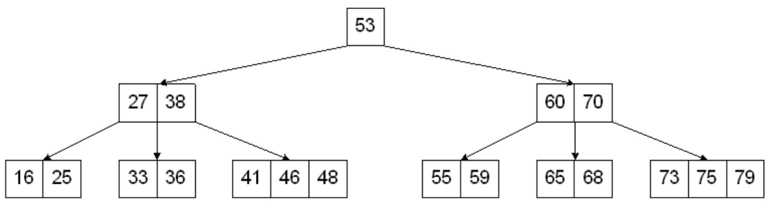


⟨ 4 ⟩ (16 pts) Define each of the following in a short yet precise manner:

(a) Exclusive-or (XOR) doubly linked list
(b) Tail recursion
(c) Average-case running time of an algorithm
(d) Abstract data type
(e) Expression tree
(f) Double rotation in rebalancing an AVL tree
(g) Color-swap in rebalancing a red-black tree
(h) Top-down red-black tree

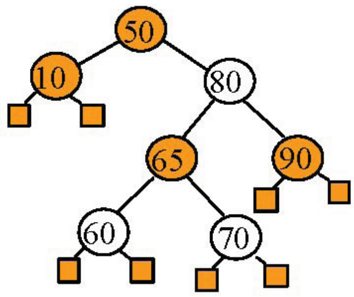
⟨ 5 ⟩ (14 pts) Given the 2-3-4 tree below, answer the following question:

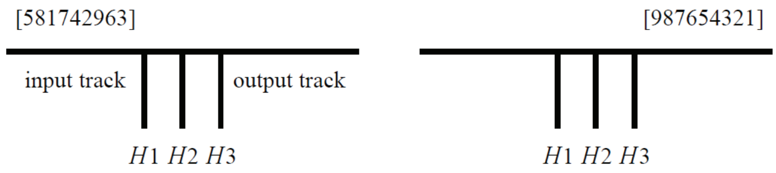
(a) (5 pts) Draw the equivalent red-black tree. Clearly mark the red and the black nodes.
(b) (9 pts) Delete 16, 25, 27, 33, 36 (in the given order) from the 2-3-4 tree. Show the resulting tree. Show your derivation in sufficient detail.



⟨ 6 ⟩ (10 pts) Given the red-black tree below, in which dark nodes are black nodes and light nodes are red nodes. First insert 71 then delete 10 on the red-black tree. Show the resulting tree, and show your derivation in sufficient detail.

⟨ 7 ⟩ (10 pts)A freight train has n railroad cars. The n cars of the freight train begin in the input track and are to end up in the output track in the order 1 through n from right to left. See the following figure in which $n = 9$; the cars are initially in the order [5, 8, 1, 7, 4, 2, 9, 6, 3] from back to front. Design a strategy to rearrange the cars using three stacks $H1, H2, H3$ so that the output becomes [9, 8, 7, 6, 5, 4, 3, 2, 1]. Describe your strategy (algorithm) in Chinese or English.





(a) Initial (b) Final

Solution: Consider the arrangement which has the cars in the input track in the order [5; 8; 1; 7; 4; 2; 9; 6; 3]. The first car, 3, cannot be moved to the output track, so it will be placed on the first holding track. Similarly, car 6 should be placed on hold. Since 3 will be moved to the output track earlier than 6, we cannot place 6 in the same holding track as 3 (otherwise 3 will be blocked). So 6 will be placed on the second holding track. Similarly, 9 should be placed in the third holding track. When we reach 2, since 2 is going to be moved to the output track before 3, we can place it in the same holding track as 3. In general, if we need to place a car k on a holding track, we will place it in the holding track whose top element is the smallest element greater than k. If no holding track has a top element greater than k, then a new holding track has to be used (if any is available - if not, then the rearrangement is impossible).

Each time a car is moved to the output track, we need to verify whether any car in the holding track can now be moved to the output track as well.