Data structures

Fall 2003

Midterm Solution

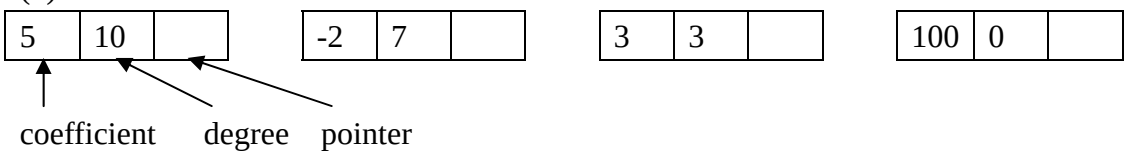
1.

(1) A collection of data along with specific operations that manipulate the data; It has nothing to do with a programming language. E.g. queue

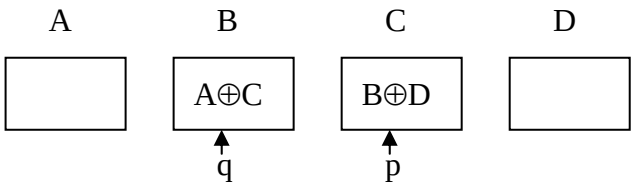
(2) Implementation of ADT. E.g. array

(3) A BST is called balanced if the height of the tree is bounded by O(log n).

(4) link list with three data filed at each node



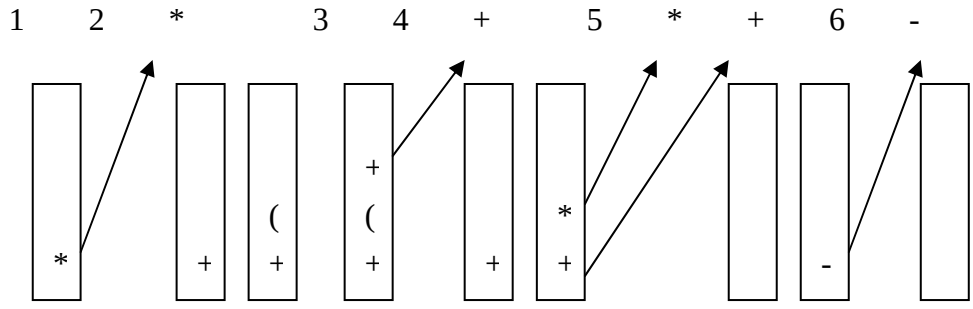
(5)



traverse back: A = (content of B) ⊕ p = (A⊕C) ⊕ C

traverse forward: D = (content of C) ⊕ q = (B⊕D) ⊕ B

2.



3.

(1) θ(1)
(2) θ(n)
(3) θ(log n)
(4) θ(log n)
(5) θ(n)
(6) θ(1)
(7) θ(n)
(8) θ(1)
(9) θ(1)
(10) θ(n)

4.

let m(h) and M(h) be the minimum and maximum number of nodes in an AVL tree of

height h. If an AVL tree with 50 nodes has height h it must be that $m(h) \le 50 \le M(h)$. Therefore, we are looking for the minimum and maximum h that satisfy the above inequalities.

As we have seen m(h) is given by the recurrence

$$m(h)=\begin{cases}1, & \text{if } h=0\\ 2, & \text{if } h=1\\ m(h-1)+m(h-2)+1, & \text{if } h>1\end{cases}$$

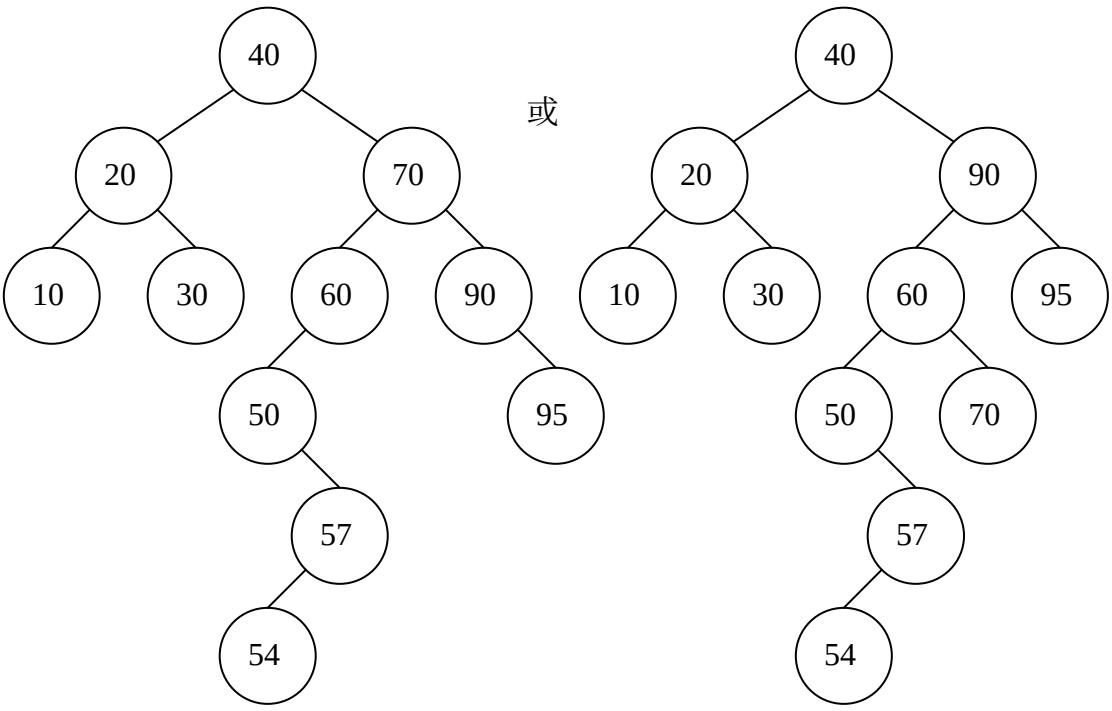
On the other hand, $M(h)=1+2+2^2+\ldots+2^h=2^{h+1}-1$. Therefore,

h	m(h)	M(h)
0	1	1
1	2	3
2	4	7
3	7	15
4	12	31
5	20	63
6	33	127
7	54	255

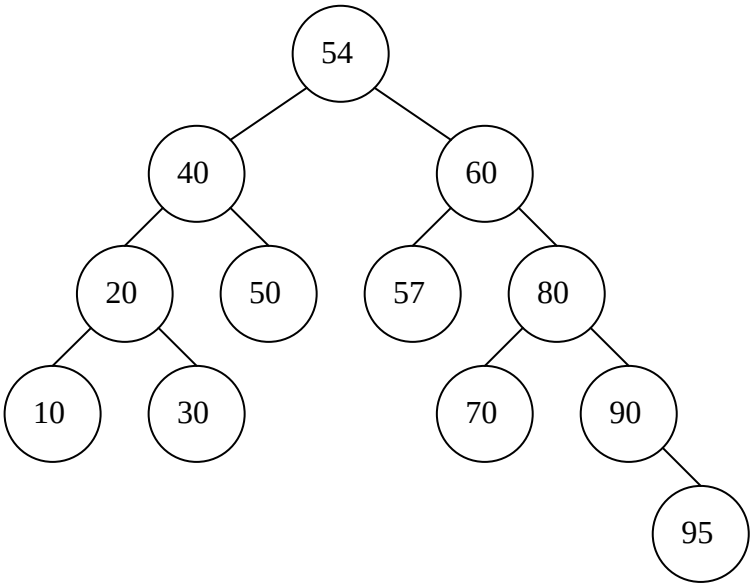
So, the minimum and maximum h that satisfy $m(h) \le 50 \le M(h)$ are 5 and 6.

5.

(1)



(2)

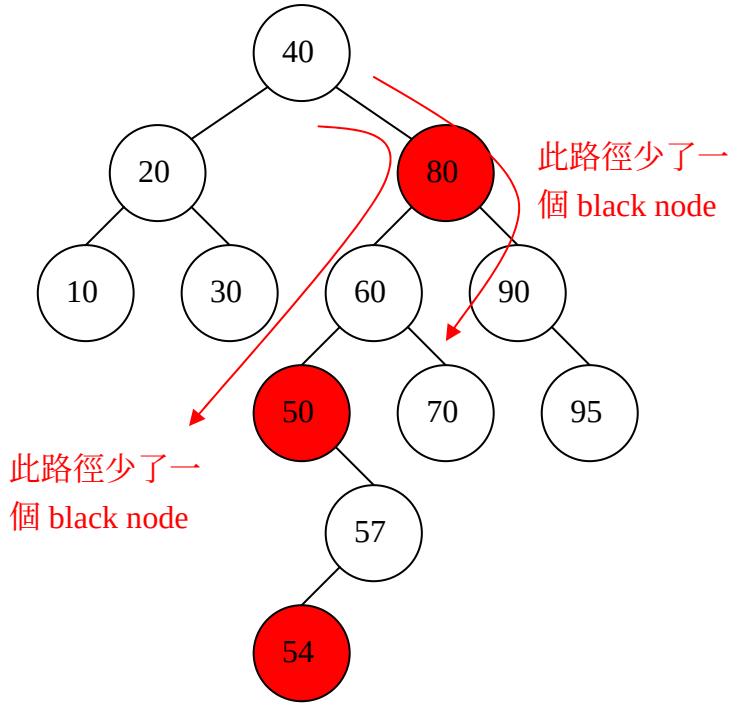


(3) preorder 40,20,10,30,80,60,50,57,54,70,90,95

inorder 10,20,30,40,50,54,57,60,70,80,90,95

postorder: 10,30,20,54,57,50,70,60,95,90,80,40

(4) 不行，因為根據 red-black tree 的定義，從 root 到任何 external node 的 path 要有相同多的 black nodes，但是在此題目中式不可能達到此要求，所以無法畫成 red-black tree

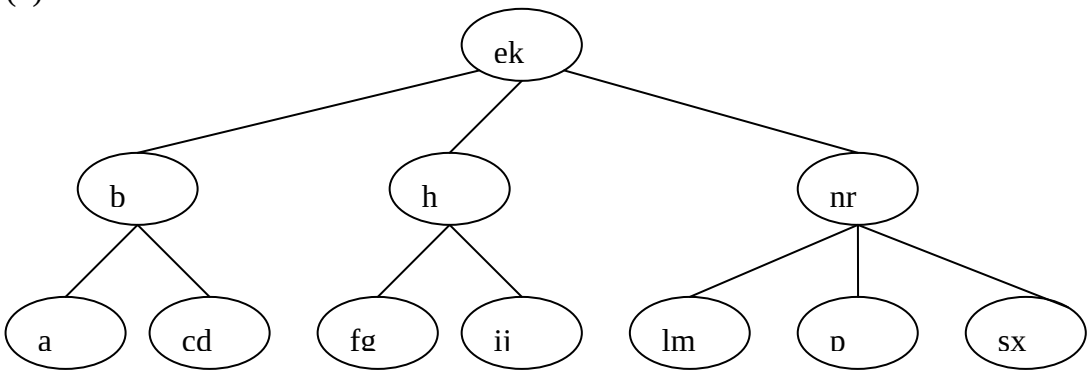


6.

index	0	1	2	3	4	5	6	7	8	9
	20	0	42	99	75	15	25	36	47	89

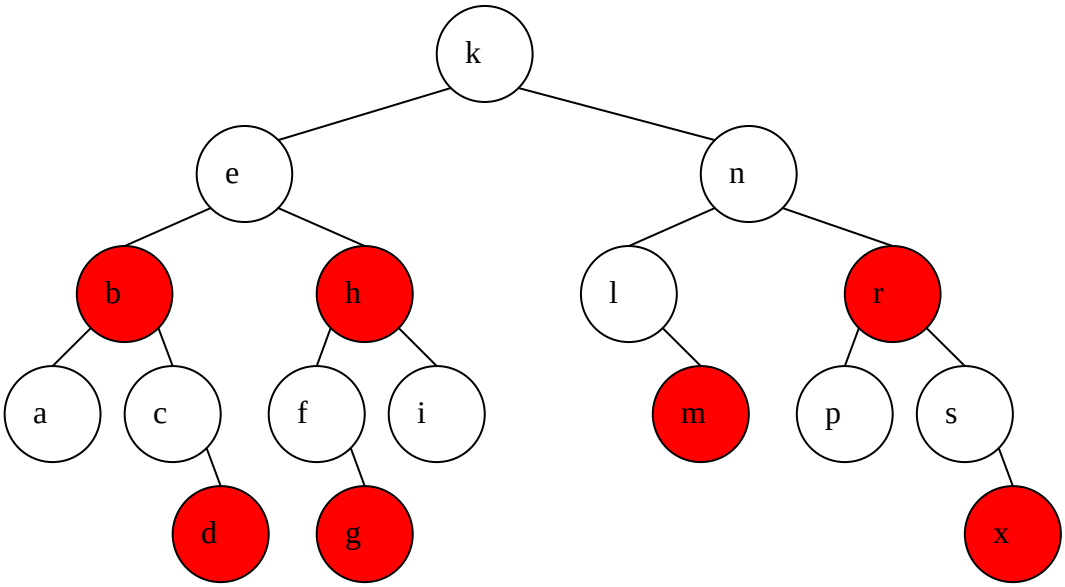
7.

(1)



(2)

before insert j



8. 2B134BB