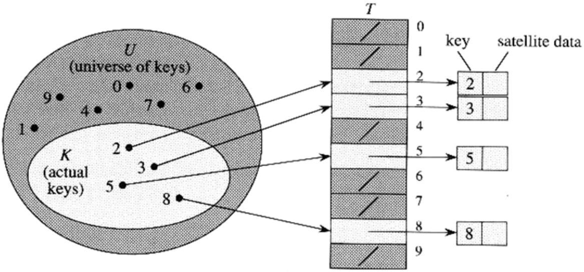
Hashing

Hash table

- Support the following operations
  - Find
  - Insert
  - Delete. (deletions may be unnecessary in some applications)
- Unlike binary search tree, AVL tree and B+-tree, the following functions cannot be done:
  - Minimum and maximum
  - Successor and predecessor
  - Report data within a given range
  - List out the data in order

Unrealistic solution

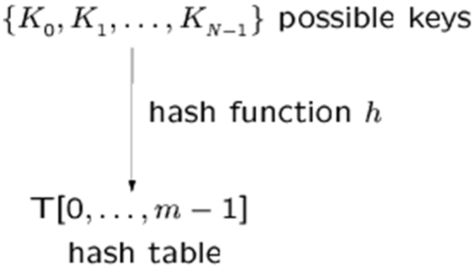
- Each position (slot) corresponds to a key in the universe of keys
  - T[k] corresponds to an element with key k
  - If the set contains no element with key k, then T[k]=NULL



Unrealistic solution

- Insert, delete and find all take O(1) (worst-case) time
- Problem:
  - The scheme wastes too much space if the universe is too large compared with the actual number of elements to be stored. E.g. student IDs are 8-digit integers, so the universe size is 10^8, but we only have about 7000 students

Hashing



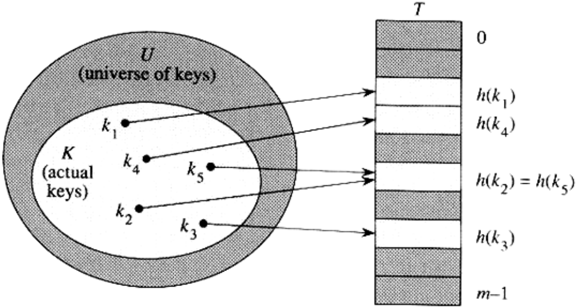
Usually, $m << N$.
$h(K_i)$ = an integer in $[0, \cdots, m-1]$ called the hash value of K_i

Example applications

- Compilers use hash tables (symbol table) to keep track of declared variables.
- On-line spell checkers. After prehashing the entire dictionary, one can check each word in constant time and print out the misspelled word in order of their appearance in the document.
- Useful in applications when the input keys come in sorted order. This is a bad case for binary search tree. AVL tree and B+-tree are harder to implement and they are not necessarily more efficient.

Hashing

- With hashing, an element of key k is stored in $T[h(k)]$



- h: hash function
  - maps the universe U of keys into the slots of a hash table $T[0, 1, ..., m-1]$
  - an element of key k hashes to slot $h(k)$
  - h(k) is the hash value of key k

Hashing

- Problem: collision
  - two keys may hash to the same slot
  - can we ensure that any two distinct keys get different cells?
    No, if $|U| > m$, where m is the size of the hash table
- Design a good hash function
  - that is fast to compute and
  - can minimize the number of collisions
- Design a method to resolve the collisions when they occur

Hash Function

- The division method
  - $h(k) = k \ mod \ m$
  - e.g. $m = 12, k = 100, h(k) = 4$

  Requires only a single division operation (quite fast)
- Certain values of m should be avoided
  - e.g. if $m = 2^p$, then $h(k)$ is just the p lowest-order bits of k; the hash function does not depend on all the bits
  - Similarly, if the keys are decimal numbers, should not set m to be a power of 10
- It's a good practice to set the table size m to be a prime number
- Good values for m: primes not too close to exact powers of 2
  - e.g. the hash table is to hold 2000 numbers, and we don't mind an average of 3 numbers being hashed to the same entry. (Choose $m = 701$)

Hash Function...

- Can the keys be strings?
- Most hash functions assume that the keys are natural numbers
  - if keys are not natural numbers, a way must be found to interpret them as natural numbers
- Method 1
  - Different permutations of the same set of characters would have the same hash value
  - If the table size is large, the keys are not distribute well. e.g. Suppose m=10007 and all the keys are eight or fewer characters long. Since ASCII value $<= 127$, the hash function can only assume values between 0 and $127 * 8 = 1016$

Hash Function...

- Method 2

a,…,z and space

27^2

```
int hash( const string & key, int tableSize )
{
    return ( key[ 0 ] + 27 * key[ 1 ] + 729 * key[ 2 ] ) % tableSize;
}
```

  - If the first 3 characters are random and the table size is $10,0007 =>$ a reasonably equitable distribution
  - Problem
    - English is not random
    - Only 28 percent of the table can actually be hashed to (assuming a table size of 10,007)

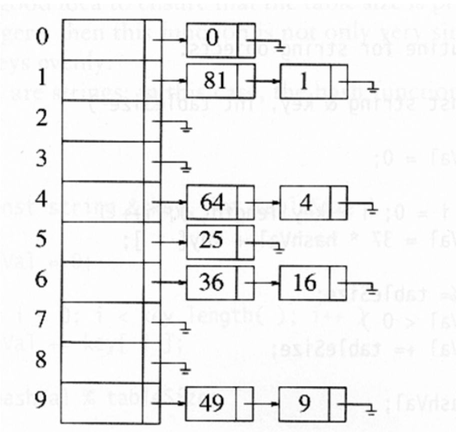
- Method 3

$$\sum_{i=0}^{KeySize-1} Key[KeySize-i-1]*37^i$$

  - Compute
  - involves all characters in the key and be expected to distribute well

Collision Handling: (1) Separate Chaining

- Instead of a hash table, we use a table of linked list
- keep a linked list of keys that hash to the same value



$h(k) = k \; mod \; 10$

Separate Chaining

- To insert a key K
  - Compute $h(K)$ to determine which list to traverse
  - If $T[h(K)]$ contains a null pointer, initiatize this entry to point to a linked list that contains K alone.
  - If $T[h(K)]$ is a non-empty list, we add K at the beginning of this list.
- To delete a key K
  - compute $h(K)$, then search for K within the list at $T[h(K)]$. Delete K if it is found.

Separate Chaining

- Assume that we will be storing n keys. Then we should make m the next larger prime number. If the hash function works well, the number of keys in each linked list will be a small constant.
- Therefore, we expect that each search, insertion, and deletion can be done in constant time.
- Disadvantage: Memory allocation in linked list manipulation will slow down the program.
- Advantage: deletion is easy.

Collision Handling: (2) Open Addressing

- Open addressing:
  - relocate the key K to be inserted if it collides with an existing key. That is, we store K at an entry different from T[h(K)].
- Two issues arise
  - what is the relocation scheme?
  - how to search for K later?
- Three common methods for resolving a collision in open addressing
  - Linear probing
  - Quadratic probing
  - Double hashing

Open Addressing

- To insert a key K, compute $h_0(K)$. If $T[h_0(K)]$ is empty, insert it there. If collision occurs, probe alternative cell $h_1(K), h_2(K), ...$ until an empty cell is found.
- $h_i(K) = (hash(K) + f(i)) \; mod \; m$, with $f(0) = 0$
  (f: collision resolution strategy)

Linear Probing

- $f(i) = i$
  - cells are probed sequentially (with wraparound)
    $h_i(K) = (hash(K) + i) \ mod \ m$
- Insertion
  - Let K be the new key to be inserted. We compute $hash(K)$
  - For $i = 0$ to $m - 1$
    - ★ compute $L = (hash(K) + I) \ mod \ m$
    - ★ $T[L]$ is empty, then we put K there and stop.
  - If we cannot find an empty entry to put K, it means that the table is full and we should report an error.

Linear Probing

- $h_i(K) = (hash(K) + i) \ mod \ m$
- E.g, inserting keys 89, 18, 49, 58, 69 with $hash(K) = K \ mod \ 10$
  - To insert 58, probe T[8], T[9], T[0], T[1]
  - To insert 69, probe T[9], T[0], T[1], T[2]

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1					58	58
2						69
3						
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89

Primary Clustering

- We call a block of contiguously occupied table entries a cluster
- On the average, when we insert a new key K, we may hit the middle of a cluster. Therefore, the time to insert K would be proportional to half the size of a cluster. That is, the larger the cluster, the slower the performance.
- Linear probing has the following disadvantages:
  - Once $h(K)$ falls into a cluster, this cluster will definitely grow in size by one. Thus, this may worsen the performance of insertion in the future.
  - If two cluster are only separated by one entry, then inserting one key into a cluster can merge the two clusters together. Thus, the cluster size can increase drastically by a single insertion. This means that the performance of insertion can deteriorate drastically after a single insertion.
  - Large clusters are easy targets for collisions.

Quadratic Probing

- $f(i) = i^2$
- $h_i(K) = (hash(K) + i^2)\ mod\ m$
- E.g., inserting keys 89, 18, 49, 58, 69 with $hash(K) = K\ mod\ 10$
  - To insert 58, probe T[8], T[9], T[(8+4) mod 10]
  - To insert 69, probe T[9], T[(9+1) mod 10], T[(9+4) mod 10]

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1						
2					58	58
3						69
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89

Quadratic Probing

- Two keys with different home positions will have different probe sequences
  - e.g. m=101, h(k1)=30, h(k2)=29
  - probe sequence for k1: 30, 30+1, 30+4, 30+9
  - probe sequence for k2: 29, 29+1, 29+4, 29+9
- If the table size is prime, then a new key can always be inserted if the table is at least half empty (see proof in text book)
- Secondary clustering
  - Keys that hash to the same home position will probe the same alternative cells
  - Simulation results suggest that it generally causes less than an extra half probe per search
  - To avoid secondary clustering, the probe sequence need to be a function of the original key value, not the home position

Double Hashing

- To alleviate the problem of clustering, the sequence of probes for a key should be independent of its primary position => use two hash functions: $hash()$ and $hash2()$
- $f(i) = i * hash2(K)$
  - E.g. $hash2(K) = R - (K \; mod \; R)$, with R is a prime smaller than m

Double Hashing

- $h_i(K) = (hash(K) + f(i))\ mod\ m; hash(K) = K\ mod\ m$
- $f(i) = i * hash2(K); hash2(K) = R - (K\ mod\ R),$
- Example: m=10, R = 7 and insert keys 89, 18, 49, 58, 69
  - To insert 49, hash2(49)=7, 2nd probe is T[(9+7) mod 10]
  - To insert 58, hash2(58)=5, 2nd probe is T[(8+5) mod 10]
  - To insert 69, hash2(69)=1, 2nd probe is T[(9+1) mod 10]

	Empty Table	After 89	After 18	After 49	After 58	After 69
0						69
1						
2						
3					58	58
4						
5						
6				49	49	49
7						
8			18	18	18	18
9		89	89	89	89	89

Choice of hash2()

- $Hash2()$ must never evaluate to zero
- For any key K, $hash2(K)$ must be relatively prime to the table size m. Otherwise, we will only be able to examine a fraction of the table entries.
  - E.g., if $hash(K) = 0$ and $hash2(K) = m/2$, then we can only examine the entries $T[0], T[m/2]$, and nothing else!
- One solution is to make m prime, and choose R to be a prime smaller than m, and set $hash2(K) = R - (K \, mod \, R)$
- Quadratic probing, however, does not require the use of a second hash function
  - likely to be simpler and faster in practice

Deletion in open addressing

- Actual deletion cannot be performed in open addressing hash tables
  - otherwise this will isolate records further down the probe sequence
- Solution: Add an extra bit to each table entry, and mark a deleted slot by storing a special value DELETED (tombstone)