PODEA: POwer Delivery Efficient Analysis with Realizable Model Reduction*

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ABSTRACT

The huge number of independent sources of power delivery system prevents the use of traditional model reduction algorithms due to the port domination nature. This paper presents an innovative RC model reduction method, PODEA, to analyze RC linear circuits with many dynamic independent sources. Based on multi-port Norton theorem and model order reduction techniques, we develop and apply current source transformation algorithm to transform attached current sources from one node to its neighboring nodes. Since there is no source attached, general RC reduction algorithms can be applied to eliminate the node. Experimental results demonstrate the efficiency and accuracy of the proposed PODEA algorithm. With linear running time, for case with over 50,000 nodes, our reduction method only takes about 0.6 seconds while maintain with 1% of error and 88% reduction ratio.

1. INTRODUCTION

Due to the higher power dissipation, lower supply voltage and faster operation frequency of VLSI chips nowadays, power grid analysis has become one of the designer's high-priority concerns. Power delivery noises, which include IR-drop, Ldi/dt-drop and resonance, may degrade the system performance or even affect its functionality. Hence extensive analyses of power delivery system are required to ensure that they meet the targeted performance and reliability goals [1] [2] [3].

Size explosion is one of the major obstacles to analyze power delivery system efficiently. Typically the power delivery network has 1 million to 100 million nodes and lots of distributed voltage and current sources. The tremendous amount of power delivery elements prevents the general-purpose circuit simulators such as SPICE to fulfill the demanding task in a timely manner.

Model order reduction has been extensively studied during the last decade to overcome this obstacle, see [4] [5] [6] [7] [8] [9] [10]. The major difficulty of applying model order reduction algorithms to power delivery circuits is the size explosion of port number since the performance and reduction ratio of most reduction algorithms, such as PRIMA [6], are strongly dependent on the number of ports. Each current or voltage source in power delivery system has to be treated as a port in PRIMA and hence it could easily exceed more than millions of ports which makes it computationally prohibitive. [9] and [10] propose Gauss elimination type algorithms to eliminate the insignificant nodes from its time constant. Albeit their great efficiency and accuracy, [9] and [10] can not handle circuits with independent sources such as power delivery circuits.

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In this paper, we propose an efficient RC model reduction method, PODEA, with emphasis on power delivery system analysis. We develop CTRAN algorithm, which integrates multi-port Norton theorem with finite time Piece-Wise-Linear (PWL) modeling of independent sources, to transform current source on one node to its neighboring nodes. Once the attached current source is transformed, RC reduction algorithms such as [7] [8] [9] [10] can be applied to eliminate the node. The CTRAN algorithm is easy to calculate and has controllable accuracy. It cooperates well with Gaussian elimination type reduction algorithms which reduce most circuits in linear time.

In the following section, we will present our PODEA reduction method and explain CTRAN algorithm in details. Then we will discuss the computation complexity of PODEA and illustrate it with several examples and report the experimental results. Last we will make some conclusion remarks.

2. POWER DELIVERY ANALYSIS MICROMODELING ALGORITHM

In this section we will present our PODEA algorithm for reducing RC circuits with many internal current sources.

Given an RC circuit with many internal current sources, our reduction method consists of three steps. First, we partition the circuit into two parts: part one includes nodes that connect no current sources, and part two includes nodes with attached current sources. The second step is to reduce nodes in part one using traditional RC model reduction algorithms, such as [7] [8] [9] [10]. In the third step, two phases are applied to each node in part two: first we use CTRAN algorithm to transform the attached current source to its neighboring nodes, then Gaussian elimination type algorithms such as TICER can be applied to reduce the node. Table 2-1 presents the outline of PODEA. Figure 2-1 shows the typical reduction procedure in step three.

Algorithm: PODEA, POwer Delivery Efficient Analysis
Step 1: Partition the given circuit into two parts: part one includes nodes which connect no current sources; part two includes nodes with attached current sources.
Step 2: Reduce part one using traditional realizable RC model order reduction algorithms.
Step 3: For each node in part two, the following two phases are applied to eliminate the node.
- **3-1:** Apply CTRAN to transform its attached current source to the neighboring nodes.
- **3-2:** Apply Gaussian elimination type model order reduction algorithms to eliminate the node.

Table 2-1. Outline of PODEA algorithm

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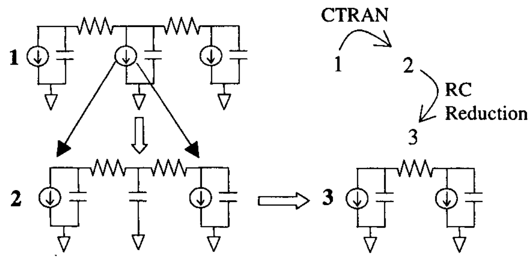


Figure 2-1. Typical reduction procedure in step three.

In the following subsections, we will focus on deriving CTRAN algorithm and explaining how CTRAN can be used to transform current sources. First, we will use an m-terminal star network to model the basic structure in power delivery system and derive the Norton equivalent current sources on its terminals. After that we introduce the finite time PWL waveform of internal current sources. Based on this background, we will derive and present CTRAN algorithm in details. In the last subsection, computation complexity of PODEA is discussed.

2.1 M-terminal Norton Equivalent Circuit and PWL Current Sources

A basic m-terminal star network with central node N can be used to model the basic structure of RC power delivery system, see Figure 2-2 (a). A central current source I_N is attached on central node N. The i^{th} branch consisting of a conductance g_i and a capacitance c_i joins the . . terminal to the central node N.

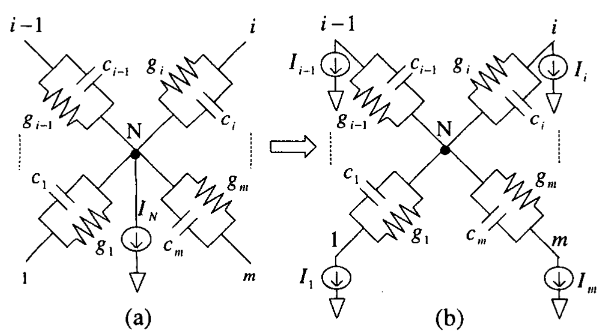


Figure 2-2. An m-terminal star network and its Norton equivalent circuit.

We can obtain the Norton equivalent current sources on each terminal by turning on the central current source and grounding all the terminals. Then the central current source can be replaced by equivalent terminal current sources according to substitution theorem, see Figure 2-2 (b). The equivalent current source on the i^{th} terminal is given by:

$$I_i = \frac{g_i + sc_i}{G + sC} I_N \qquad (2\text{-}1)$$

where $G = \sum_{i=1}^{m} g_i$, $C = \sum_{i=1}^{m} c_i$ are total conductance and capacitance of central node N respectively. Equation (2-1) gives us a way to calculate equivalent current sources. However we also need a method to model the internal current sources. This modeling method should have controllable accuracy and by using this model, Equation (2-1) is easy to calculate.

Finite time PWL sources can fulfill the task of modeling most types of waveforms with controllable accuracy. Given two PWL sources with different set of time delays, it's easy to add them together. Assume I_N is a finite time PWL current source represented by delayed steps and ramps as shown in Figure 2-3. For the i^{th} segment, α_i^N , γ_i^N and τ_i^N stand for the initial value, slope and time delay respectively.

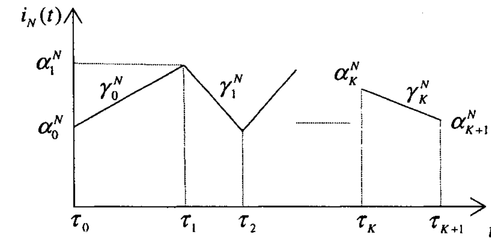


Figure 2-3. Piece wise linear waveform of $i_N(t)$.

$i_N(t)$ can be represented in the time domain as:

$$i_N(t) = \sum_{j=0}^{K} \{[\alpha_j^N + \gamma_j^N (t-\tau_j)]E(t-\tau_j)$$
$$-[\alpha_{j+1}^N + \gamma_j^N (t-\tau_{j+1})]E(t-\tau_{j+1})\} \qquad (2\text{-}2)$$

Laplace transformation in s-domain is given by:

$$I_N = \sum_{j=0}^{K} [(\frac{\alpha_j^N}{s} + \frac{\gamma_j^N}{s^2})e^{-\tau_j s} - (\frac{\alpha_{j+1}^N}{s} + \frac{\gamma_j^N}{s^2})e^{-\tau_{j+1} s}] \qquad (2\text{-}3)$$

With Equation (2-1) and representation of I_N in Equation (2-3), we can develop our CTRAN algorithm to calculate Norton equivalent current sources.

2.2 CTRAN: An Efficient Way for Transforming the Internal Current Sources

In this subsection, we introduce CTRAN algorithm which transforms current source on one node to its neighboring nodes.

Substitute Equation (2-3) to Equation (2-1), we get the equivalent current source on the i^{th} terminal in s-domain:

$$I_i = (\frac{g_i + sc_i}{G + sC}) \sum_{j=0}^{K} [(\frac{\alpha_j^N}{s} + \frac{\gamma_j^N}{s^2})e^{-\tau_j s} - (\frac{\alpha_{j+1}^N}{s} + \frac{\gamma_j^N}{s^2})e^{-\tau_{j+1} s}] \qquad (3\text{-}1)$$

First consider the simplest scenario in which the central node is a resistive node (node with only resistors connected to it). In this case, Equation (3-1) equals to:

$$I_i = \sum_{j=0}^{K} [(\frac{\alpha_j^i}{s} + \frac{\gamma_j^i}{s^2})e^{-\tau_j s} - (\frac{\alpha_{j+1}^i}{s} + \frac{\gamma_j^i}{s^2})e^{-\tau_{j+1} s}] \qquad (3\text{-}2)$$

where

$$\alpha_j^i = \frac{g_i}{G}\alpha_j^N,\ \gamma_j^i = \frac{g_i}{G}\gamma_j^N \tag{3-3}$$

Equation (3-3) can be translated into a procedure for calculating the equivalent current sources on neighboring nodes. The equivalent current source on the i^{th} neighboring node can be obtained by multiplying the initial values of central current source by g_i / G. In the case that the central node is a capacitive node, the equivalent current source on the i^{th} neighboring node equals to multiple each initial value of central current by c_i / C.

Next we consider the general case. To get the equivalent current source on the i^{th} terminal, $i_i(t)$, we need to calculate its values at different time delays. Equation (3.1) can be shown as follows:

$$I_i = \sum_{j=0}^{K}[(\frac{\mu_{j,1}^i}{s} + \frac{v_j^i}{s^2} + \frac{\omega_{j,1}^i}{s+G/C})e^{-\tau_j s}$$

$$-(\frac{\mu_{j,2}^i}{s} + \frac{v_j^i}{s^2} + \frac{\omega_{j,2}^i}{s+G/C})e^{-\tau_{j+1} s} \tag{3-4}$$

where

$$v_j^i = \frac{g_i}{G}\gamma_j^N$$

$$\mu_{j,1}^i = \frac{G(g_i\alpha_j^N + c_i\gamma_j^N) - Cg_i\gamma_j^N}{G^2}$$

$$\mu_{j,2}^i = \frac{G(g_i\alpha_{j+1}^N + c_i\gamma_j^N) - Cg_i\gamma_j^N}{G^2}$$

$$\omega_{j,1}^i = \frac{G^2c_i\alpha_j^N + C^2g_i\gamma_j^N - GC(g_i\alpha_j^N + c_i\gamma_j^N)}{G^2C}$$

$$\omega_{j,2}^i = \frac{G^2c_i\alpha_{j+1}^N + C^2g_i\gamma_j^N - GC(g_i\alpha_{j+1}^N + c_i\gamma_j^N)}{G^2C} \tag{3-5}$$

Inverse Laplace transformation of Equation (3-4) is

$$i_i(t) = \sum_{j=0}^{K}\{[\mu_{j,1}^i + v_j^i(t-\tau_j) + \omega_{j,1}^i e^{-\frac{G}{C}(t-\tau_j)}]E(t-\tau_j)$$

$$-[\mu_{j,2}^i + v_j^i(t-\tau_{j+1}) + \omega_{j,2}^i e^{-\frac{G}{C}(t-\tau_{j+1})}]E(t-\tau_{j+1})\} \tag{3-6}$$

Also we notice the following equations hold for parameters in Equations (3-4) and (3-6):

$$\mu_{j,1}^i + v_j^i\Delta\tau_{j+1,j} = \mu_{j,2}^i \tag{3-7}$$

$$\omega_{j+1,1}^i - \omega_{j,2}^i = \kappa^i\Delta\gamma_{j+1} \tag{3-8}$$

$$\mu_{j+1,1}^i + \omega_{j+1,1}^i = \mu_{j,2}^i + \omega_{j,2}^i \tag{3-9}$$

where $\kappa^i = (Cg_i - Gc_i)/G^2$, $\Delta\tau_{m,n} = \tau_m - \tau_n$, $\Delta\gamma_{j+1}^N = \gamma_{j+1}^N - \gamma_j^N$.

In order to calculate $i_i(\tau_j)$ using Equation (3-6), we need to proof $i_i(t)$ is continuous at τ_j because step function $E(t-\tau_j)$ has no definition at τ_j. This can be done by showing that $i_i(\tau_j^+) = i_i(\tau_j^-)$. For example, at τ_1:

$$i_i(\tau_1^-) = \mu_{0,1}^i + v_0^i\Delta\tau_{1,0} + \omega_{0,1}^i e^{-\frac{G}{C}\Delta\tau_{1,0}} = \mu_{0,2}^i + \omega_{0,1}^i e^{-\frac{G}{C}\Delta\tau_{1,0}}$$

$$i_i(\tau_1^+) = \mu_{1,1}^i + \omega_{1,1}^i - \omega_{0,2}^i + \omega_{0,1}^i e^{-\frac{G}{C}\Delta\tau_{1,0}} = \mu_{0,2}^i + \omega_{0,1}^i e^{-\frac{G}{C}\Delta\tau_{1,0}}$$

It follows $i_i(\tau_1^+) = i_i(\tau_1^-)$. $i_i(t)$ can be proven to be continuous in time range from τ_0^+ to τ_{K+1}^- and hence we can calculate α_j^i that is the exact value of $i_i(t)$ at τ_j by calculating $i_i(\tau_j^-)$ except at τ_0. After indulging in some algebra, we get the following formula for calculating $i_i(t)$ at different time delays.

For $t = \tau_0$, $\alpha_0^i = \frac{c_i\alpha_0^N}{C}$ (3-10)

For $t = \tau_j, j = 1, \cdots, K+1$,

$$\alpha_j^i = \mu_{j-1,2}^i + \omega_{0,1}^i e^{-\frac{G}{C}\Delta\tau_{j,0}} + \kappa^i\sum_{k=1}^{j-1}\Delta\gamma_k^N e^{-\frac{G}{C}\Delta\tau_{j,k}} \tag{3-11}$$

The CTRAN algorithm is summarized in Table 2-2.

Algorithm: CTRAN: Current Transformation Algorithm
Input: Central current source $I_N = \{\alpha_i^N, \tau_i\}_{i=0}^{K+1}$
G, total conductance; C, total capacitance
Conductor and capacitor on branches, $\{g_i, c_i\}_{i=1}^M$
M, the number of branches
Output: $I_i = \{\alpha_{0,\cdots,K+1}^i, \tau_{0,\cdots,K+1}\}_{i=1}^M$
Begin
$\sigma = -G/C$
for j = 0 : K
$\gamma_j^N = (\alpha_{j+1}^N - \alpha_j^N)/(\tau_{j+1}^N - \tau_j^N)$
end
for i = 1 : M
$\kappa = (Cg_i - Gc_i)/G^2$
$\omega = \frac{G^2c_i\alpha_0^N + C^2g_i\gamma_0^N - GC(g_i\alpha_0^N + c_i\gamma_0^N)}{G^2C}$
$\alpha_0^i = c_i\alpha_0^N / C$
for m = 1 : K+1
$\mu = \frac{G(g_i\alpha_m^N + c_i\gamma_{m-1}^N) - Cg_i\gamma_{m-1}^N}{G^2}$
$\alpha_m^i = \mu + \omega e^{\sigma(\tau_m - \tau_0)} + \kappa\sum_{n=1}^{m-1}(\gamma_n^N - \gamma_{n-1}^N)e^{\sigma(\tau_m - \tau_n)}$
end
end
End

Table 2-2. CTRAN: current transformation algorithm

In practice, we can simplify calculation by eliminating some exponential terms in Equation (3-11) according to the value of G/C and the time interval between different time delays. For example, if $G >> C$, we can expect the exponential terms in

Equation (3-11) decay very fast. In this case, the following formula can be used to approximate Equation (3-11):

$$\alpha_j^i \approx \frac{g_i \alpha_j^N + c_i \gamma_{j-1}^N}{G} \qquad (3\text{-}12)$$

Hence accuracy is controllable in the process of current source transformation.

2.3 Computation Complexity of PODEA

Our PODEA reduction method has linear running time and controllable accuracy. The running time of CTRAN is proportional to the number of neighboring nodes and the running time of Gaussian elimination type RC reduction is $O(n)$ as reported in [10]. Hence the total running time $O(\alpha n)$ can be achieved, where n is the number of eliminated nodes, and α is the average number of their neighboring nodes.

3. EXPERIMENTAL RESULTS

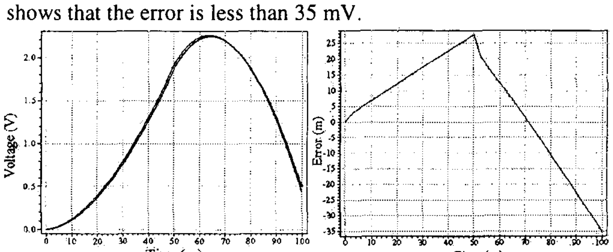
In this section, we illustrate our PODEA power delivery analysis algorithm with couple of examples. The test circuits are randomly generated RC tree. The value range of R is from 0.1 Ω to 1 Ω and C is from *1fF* to *1pF*. The proposed algorithm is implemented in C language and run on a PIII 900*MHz* machine with 256*MB* memory.

Table 3-1 lists the simulation results of different circuits. For the 51,205-node circuit, the reduction time is about 0.6 secs. The simulation speedup before and after reduction is at least 59X.

Circuit Size		Reduction Time (secs)	Percent Error	Run Time B/A Reduction (secs)		Reduction Ratio
Nodes	Sources					
50,002	30,310	0.561	1.0%	2878	48 (59X)	87.6%
51,205	32,124	0.570	1.1%	3064	49 (62X)	87.6%
65,371	32,534	0.691	N/A	N/A	72	87.4%
72,938	54,765	0.822	N/A	N/A	93	88.7%
81,125	61,205	1.092	N/A	N/A	117	87.9%
120,513	91,125	1.632	N/A	N/A	218	88.6%

Table 3-1. Some experimental results.

Figure 3-1 (a) plots the SPICE simulation of the 51,205-node circuit before and after reduction; (b) shows the error curve. It shows that the error is less than 35 mV.



(a) (b)

Figure 3-1. (a) Waveforms of the 51205-node circuit B/A reduction; (b) Error curve in terms of mV.

Figure 3-2 shows the running time and approximate memory consumption of different circuits. We can see from Figure 3-2, the running time and memory consumption is roughly linear proportional to the node number.

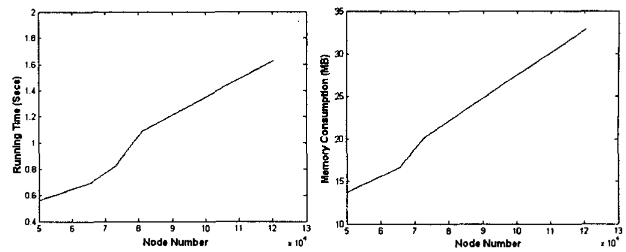


Figure 3-2. Running time and memory consumption versus the number of nodes.

4. SUMMARY

The contributions of this paper are as follows. First, we establish a realizable RC reduction method with independent current sources, PODEA, for power delivery system analysis. It has linear running time and controllable accuracy. Second we develop CTRAN algorithm to transform current sources thus traditional RC model reduction algorithms can be applied to reduce the circuits. The analysis results will help the designers to identify potential power supply noise problems and optimize various design configurations.

5. REFERENCES

[1] M. Zhao, R. V. Panda, S. S. Sapatnekar, T. Edwards, R. Chaudhry and D. Blaauw. Hierarchical analysis of power distribution networks. *DAC*, 2000.

[2] J. M. Wang and T. V. Nguyen. Extended Krylov method for reduced order analysis of linear Circuit with multiple Sources. *DAC*, 2000.

[3] Y. Cao, Y.-M. Lee, T.-H. Chen and C. C.-P. Chen. HiPRIME: hierarchical and passivity reserved interconnect macromodeling engine for RLKC power delivery. *DAC*, June 2002.

[4] L. T. Pillage and R. A. Rohrer. Asymptotic waveform evaluation for timing analysis. *IEEE Trans. Computer Aided Design*, 1990.

[5] A. Odabasioglu, M. Celik and L. T. Pillage. PRIMA: Passive reduced order interconnect macromodeling algorithm. *ICCAD*, pages 58–65, 1997.

[6] P. Rabiei and M. Pedram. Model order reduction of large circuits using balanced truncation. *DAC*, February 1999.

[7] A. Devgan and P. R. O'Brien. Realizable reduction for RC interconnect circuits. *Proc. ICCAD*, 1999.

[8] P. Ganesh and C. C.-P. Chen. RC-in RC-out model order reduction accurate up to second order moments. *ICCAD*, 1999.

[9] A. J. Genderen and N. P. Van Der Meijs. Space user's manual, Space tutorial, Space 3D capacitance extraction user's manual. Delft University of Technology, Dept. of EE, Delft, Netherlands, 1995.

[10] B. N. Sheehan. TICER: Realizable reduction of extracted RC circuits. *Proc. ICCAD*, 1999.