HiPRIME
Hierarchical and Passivity Reserved Interconnect Macromodeling Engine for RLKC Power Delivery

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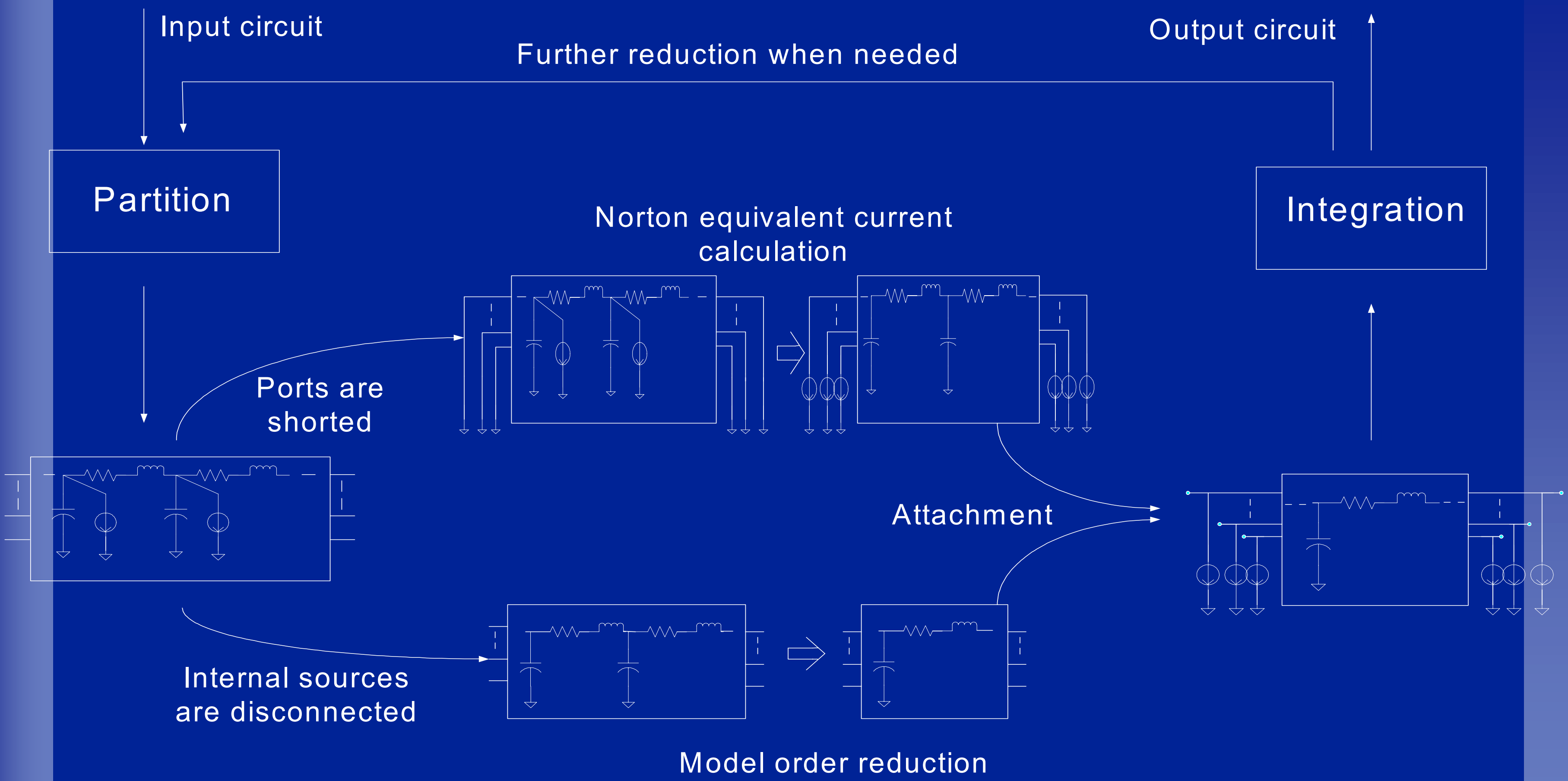
Outline

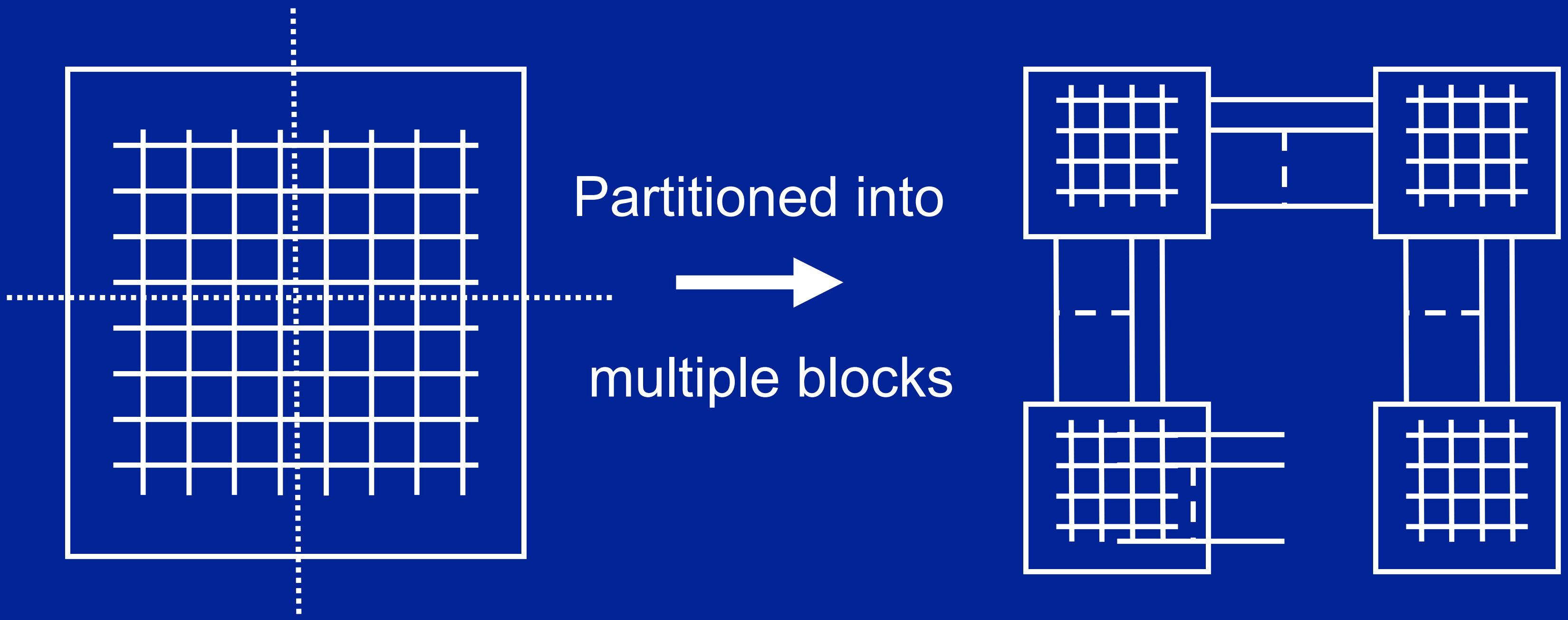
- Power grid analysis
- HiPRIME algorithm
  - Model order reduction
  - Improved Extended Krylov Subspace (IEKS)
  - Combination of macromodels
  - Higher level model order reduction
- Experimental results
- Conclusion

HiPRIME Algorithm



Circuit Partition

- **Power grid circuit is partitioned into multiple small blocks**

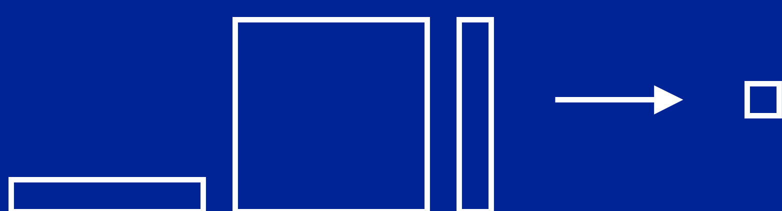


HiPRIME: Step 1

- All current sources are disconnected
- The linear part of the circuits are reduced and its passivity is reserved

$$G_i X_i + C_i \frac{d}{dt} X_i = B_i u_i \quad \xrightarrow{\text{Reduction}} \quad \tilde{G}_i \tilde{X}_i + \tilde{C}_i \frac{d}{dt} \tilde{X}_i = \tilde{B}_i u_i$$

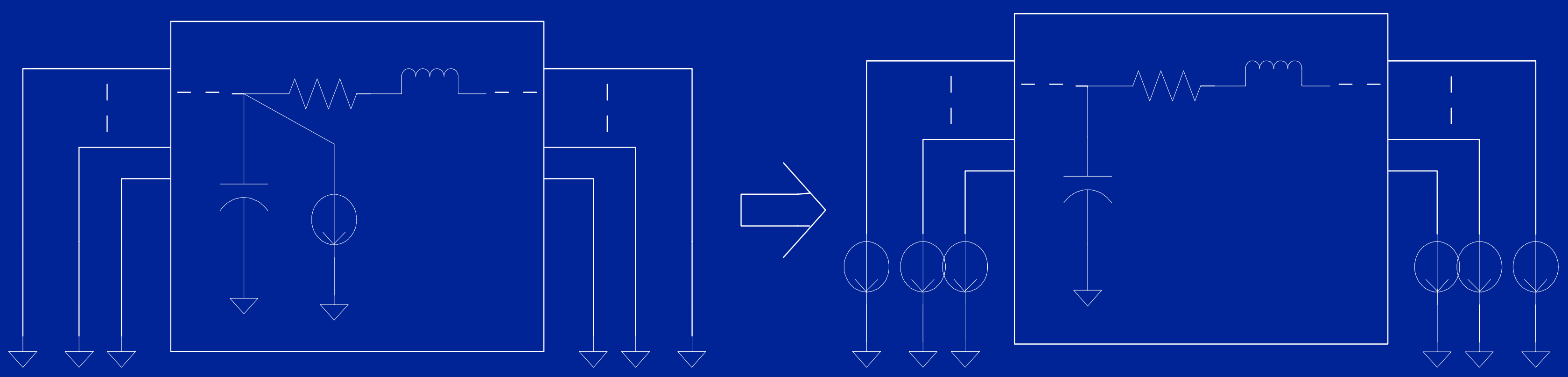
$$V^T \times G / C \times V \qquad \tilde{G} / \tilde{C}$$



$$V^T \times B \qquad \tilde{B}$$

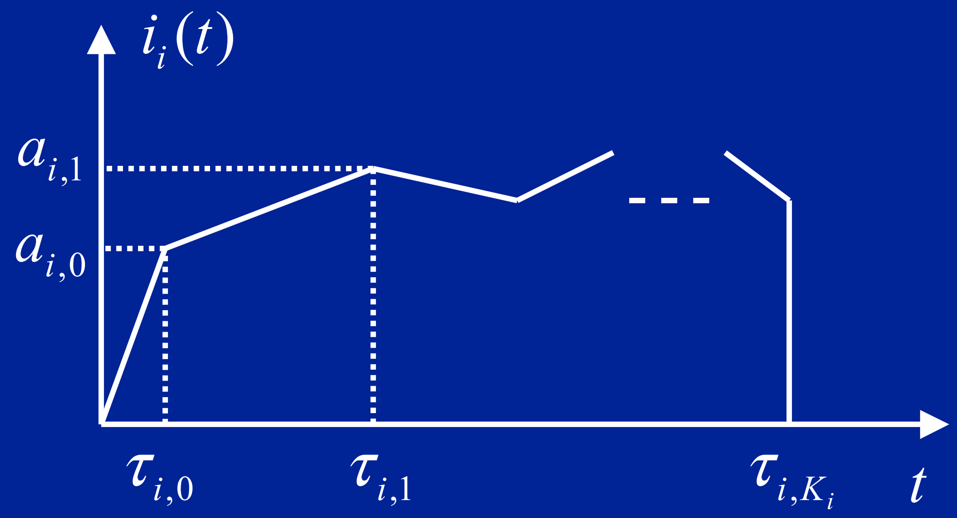
HiPRIME: Step2

- Extraction of internal currents to ports
- IEKS is used to compute port currents
- Calculation is based on the original circuit blocks



Improved Extended Krylov Subspace

- **Current source model**



Time domain representation

$$i_i(t) = \sum_{j=0}^{K_i} \left\{ \left[a_{i,j} + \gamma_{i,j}(t - \tau_{i,j}) \right] u(t - \tau_{i,j}) - \left[a_{i,j+1} + \gamma_{i,j}(t - \tau_{i,j+1}) \right] u(t - \tau_{i,j+1}) \right\}$$

IEKS (cont.)

EKS current sources frequency domain representation

$$I_i(s) = \frac{1}{s^2} \sum_{j=0}^{K_i} \left\{ \gamma_{i,j} \cdot \exp(-\tau_{i,j} s) \right\}$$

IEKS current sources frequency domain representation

$$I_i(s) = \frac{1}{s^2} \sum_{j=0}^{K_i} \left\{ s \cdot a_{i,j} \cdot \exp(-\tau_{i,j} s) + \gamma_{i,j} \cdot \exp(-\tau_{i,j} s) - s \cdot a_{i,j+1} \cdot \exp(-\tau_{i,j+1} s) - \gamma_{i,j+1} \cdot \exp(-\tau_{i,j+1} s) \right\}$$

- **Given a finite-time PWL source, IEKS calculates a moment representation with** -2^{nd} and -1^{st} **order moments to be zero.**
- **Improvement over EKS: moment switching free in moment matching iteration**

IEKS (cont.)

- Equation solution to get port currents
  - Compact description of the i^th block:

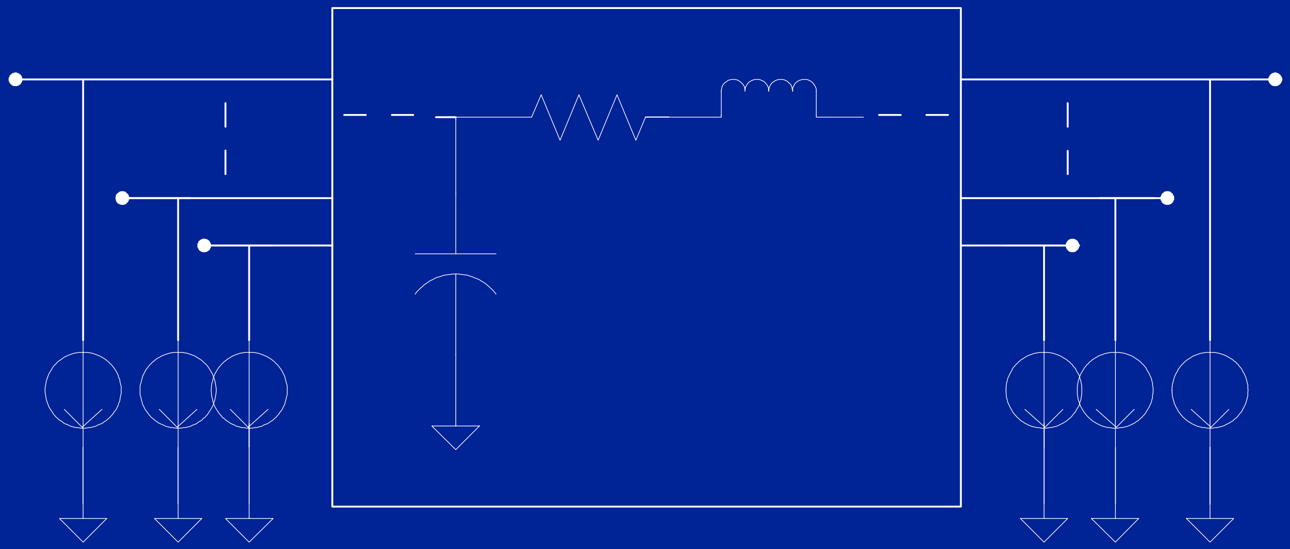
$$\widetilde{G}_i \widetilde{X}_i + \widetilde{C}_i \frac{d}{dt} \widetilde{X}_i = \left[\widetilde{B}_i \widetilde{B}_i' \right] \begin{bmatrix} 0 \\ i_{gi} \end{bmatrix}$$

  - It can be solved by standard integration methods.
  - Port current recovery:

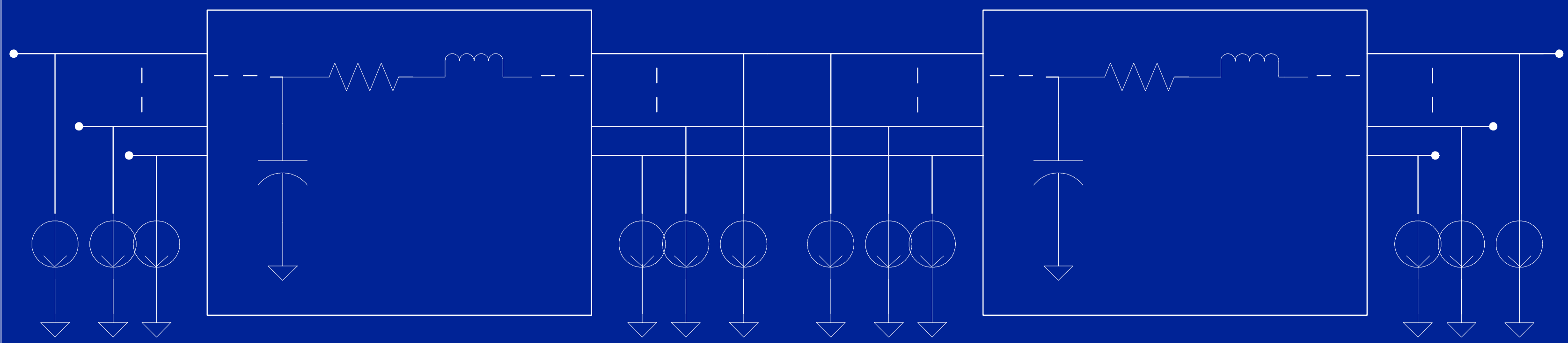
$$i_{Ni} = B^T X_i = \widetilde{B}_i^T \widetilde{X}_i$$

Multiport Norton Equivalent Model

- After Step 1 and Step 2, the macromodel of each partitioned block is established, which includes
  - Compact model of the interconnect
  - Port currents, which contribute to ports as the disconnected internal current sources



Combination of Macromodels



Formulation of the combination of the i^{th} and j^{th} blocks

$$\begin{bmatrix} \tilde{G}_i & 0 & -\tilde{B}_{ij} \\ 0 & \tilde{G}_j & -\tilde{B}_{ji} \\ \tilde{B}_{ij}^T & \tilde{B}_{ji}^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{X}_i \\ \tilde{X}_j \\ u_{ij} \end{bmatrix} + \begin{bmatrix} \tilde{C}_i & 0 & 0 \\ 0 & \tilde{C}_j & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \tilde{X}_i \\ \tilde{X}_j \\ u_{ij} \end{bmatrix} = \begin{bmatrix} \tilde{B}_{ii} & 0 & 0 & 0 \\ 0 & \tilde{B}_{jj} & 0 & 0 \\ 0 & 0 & E_i^T & E_j^T \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ i_{Ni} \\ i_{Nj} \end{bmatrix}$$

Higher-level Model Order Reduction

- **Combination of the interconnect of the ith and jth blocks**

$$\underbrace{\begin{bmatrix} \tilde{G}_i & 0 & -\tilde{B}_{ij} \\ 0 & \tilde{G}_j & -\tilde{B}_{ji} \\ \tilde{B}_{ij}^T & \tilde{B}_{ji}^T & 0 \end{bmatrix}}_{G'} \begin{bmatrix} \tilde{X}_i \\ \tilde{X}_j \\ u_{ij} \end{bmatrix} + \underbrace{\begin{bmatrix} \tilde{C}_i & 0 & 0 \\ 0 & \tilde{C}_j & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{C'} \frac{d}{dt} \begin{bmatrix} \tilde{X}_i \\ \tilde{X}_j \\ u_{ij} \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{B}_{ii} & 0 \\ 0 & \tilde{B}_{jj} \\ 0 & 0 \end{bmatrix}}_{B'} \begin{bmatrix} u_i \\ u_j \end{bmatrix}$$

- **Passivity preservation**

The system function

$$Y(s) = \tilde{B}'^T (\tilde{G}' + s\tilde{C}')^{-1} \tilde{B}'$$

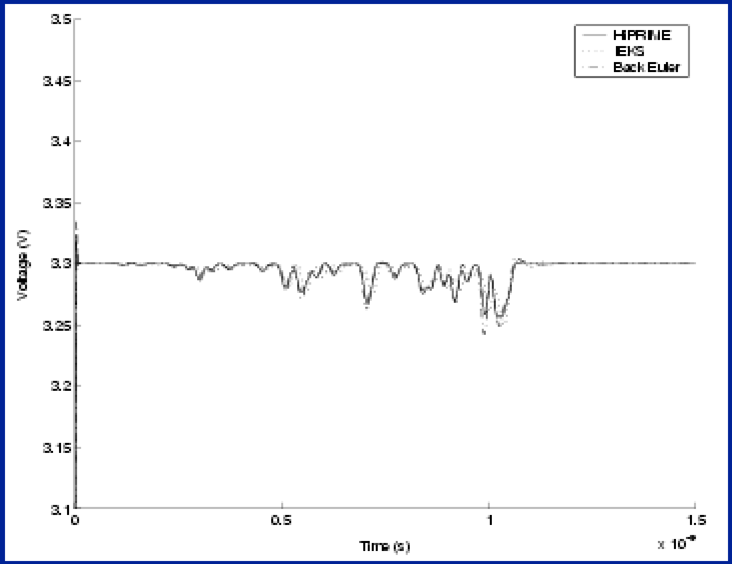
satisfies:

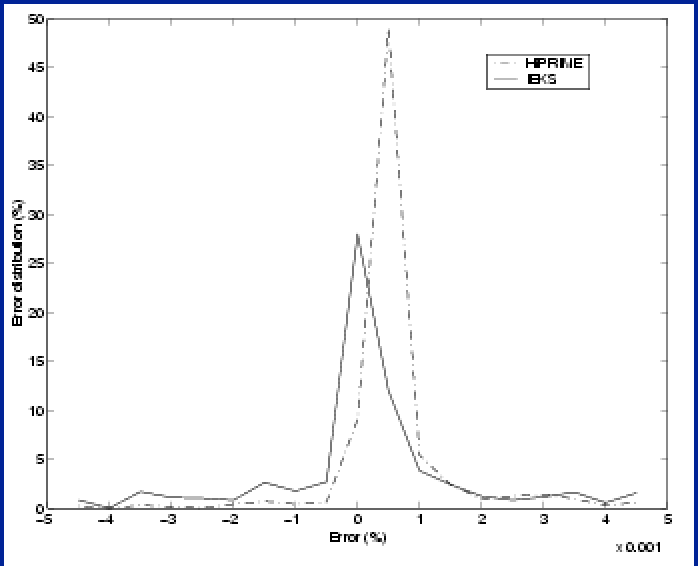
1) $Y(s^*) = Y^*(s)$ for all complex s

2) $Y(s)$ is a positive matrix, that means, $z^{*T}(Y(s) + Y^T(s^*))z \geq 0$ for all complex s satisfying $\mathrm{Re}(s) > 0$ and for any complex vector z.

Experimental Results

Accuracy (HiPRIME, IEKS and Back Euler)





Waveforms indistinguishable

50% errors are within 0.001%
Maximum error is 2%

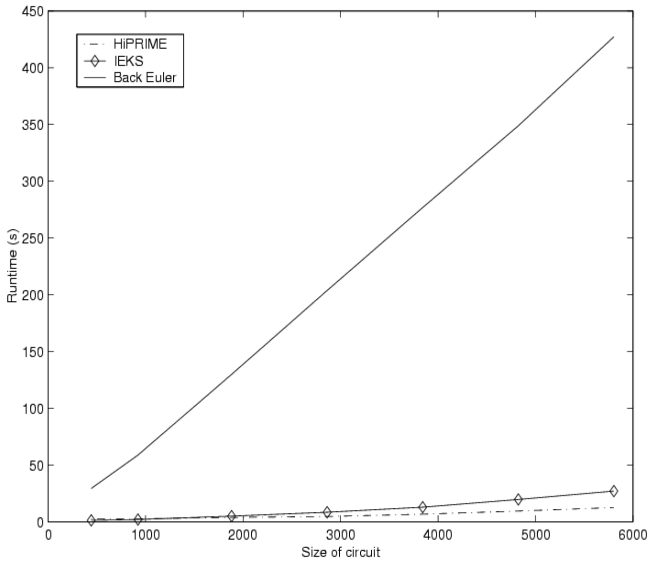
Experimental Results (cont.)

Runtime Illustration (HiPRIME, IEKS and Back Euler)

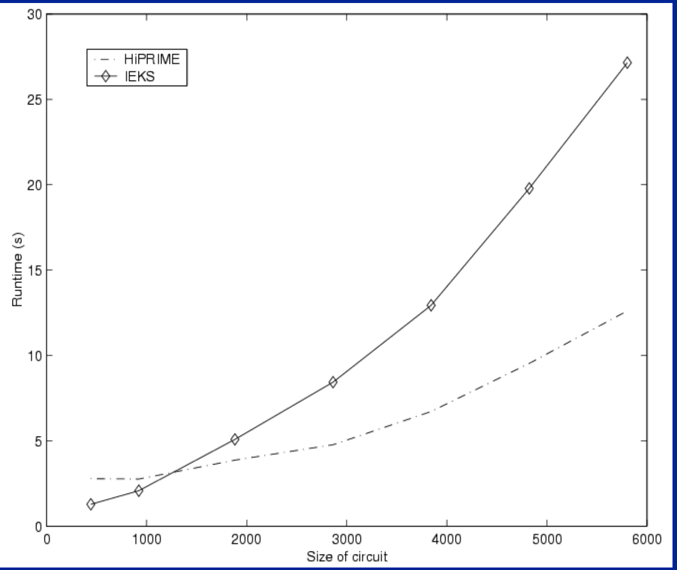
Circuit size	HiPRIME (s)	IEKS (s)	Speedup (X)	Back Euler (S)	Speed up (X)
443	2.76	1.29	0.47	29.5	10.7
1883	3.87	5.08	1.31	129.8	33.5
3843	6.72	12.94	1.92	276.9	41.2
5803	11.81	27.15	2.29	427.2	36.6

Experimental Results (cont.)

Runtime Illustration (HiPRIME, IEKS and Back Euler)



HiPRIME and IEKS outperform Back Euler



HIPRIME achieves speedup over EKS by hierarchical analysis

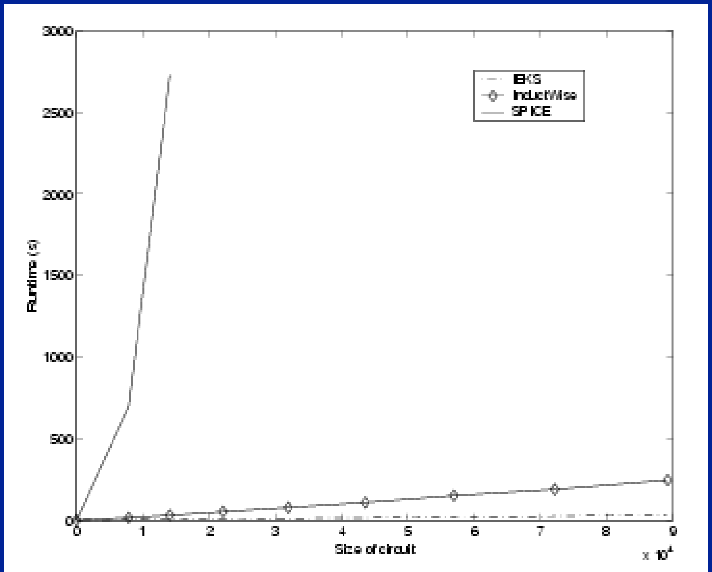
Experimental Results (cont.)

Runtime Illustration (Spice, IEKS and InductWise)

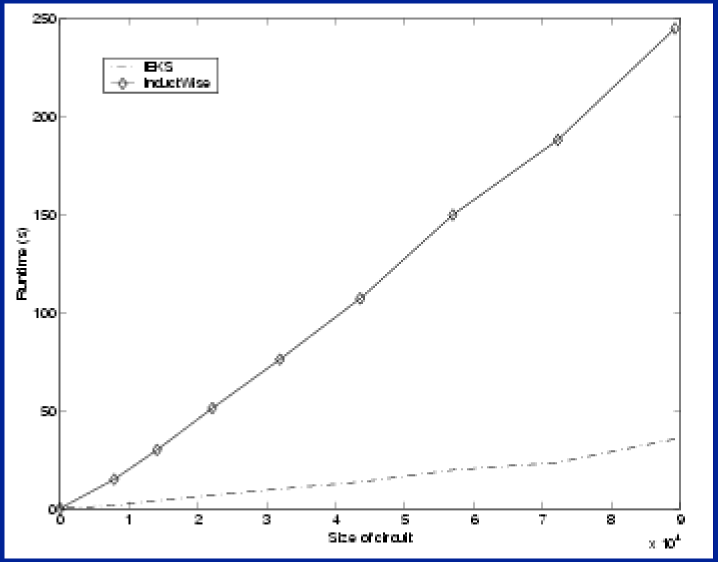
Circuit size	IEKS (s)	InductWise (s)	Speedup (X)	Spice (S)	Speed up (X)
7861	1.46	14.76	10.1	697.13	477.48
14081	3.88	29.77	7.67	2728.18	703.14
43541	13.49	107.05	7.93	--	--
89201	35.33	244.95	6.93	--	--

Experimental Results (cont.)

Runtime Illustration (Spice, IEKS and InductWise)



IEKS and InductWise outperform Spice



IEKS achieves speedup over InductWise

Conclusion

- Establish a novel hierarchical power delivery macromodeling methodology, which integrates multiple-port Norton equivalent theorem with model order reduction algorithm to generate compact and accurate model and achieve significant runtime improvement
- Enhance the EKS method such that it no longer needs to perform moment shifting for source waveform modeling
- Develop a multiple level passive model reduction algorithm and prove its passivity

Demo

Software demo at University Booth!